ANALYSIS OF DECISION-MAKING IN CLOSED-LOOP SUPPLY CHAINS

A Thesis
Presented to
The Academic Faculty

by

Chanjoo Lee

In Partial Fulfillment
of the Requirements for the Degree of
Doctor of Philosophy in the
H. Milton Stewart School of Industrial and Systems Engineering

Georgia Institute of Technology
August 2011

ANALYSIS OF DECISION-MAKING IN CLOSED-LOOP SUPPLY CHAINS

Approved by:

Professor Jane Ammons, Co-Advisor, Committee Chair
H. Milton Stewart School of Industrial and Systems Engineering
Georgia Institute of Technology

Professor Matthew Realff, Co-Advisor
School of Chemical & Biomolecular Engineering
Georgia Institute of Technology

Professor Leon McGinnis
H. Milton Stewart School of Industrial and Systems Engineering
Georgia Institute of Technology

Professor Mark Ferguson
College of Management
Georgia Institute of Technology

Associate Professor Julie Swann
H. Milton Stewart School of Industrial and Systems Engineering
Georgia Institute of Technology

Date Approved: 1 July 2011

To Jieun and Joseph

ACKNOWLEDGEMENTS

I would like to express my deepest gratitude to my thesis co-advisors, Dr. Jane Ammons and Dr. Matthew Realff, for their unrelenting support, guidance, patience, and encouragement throughout my years in Georgia Tech. Dr. Ammons provided me with the opportunities to reach my ultimate potential while helping me enjoy every challenging step in my pursuit of a PhD. Dr. Realff epitomized a passionate and enthusiastic researcher. They are the best teachers and mentors that will be my role models for my future career.

I would also like to thank my committee members: Dr. Leon McGinnis, for his insightful comments; Dr. Mark Ferguson, for helping me reframe the research problems; and Dr. Julie Swann, for her careful proofreading. This work has benefited greatly from their helpful comments and new perspectives.

Lastly, I would like to thank my parents, my brother, and my family for all their prayers, love, and encouragement. They have been an immense source of confidence and strength to me in completing this dissertation.

TABLE OF CONTENTS

LIST OF TABLES

Table

LIST OF FIGURES

Figure

SUMMARY

Closed-loop supply chains (CLSCs) that integrate the activities for reclaiming residual values in postconsumer products with the traditional forward supply chain activities are important from financial and environmental perspectives. This thesis develops models and analyses on three topics novel to the field of CLSC research with a goal of advancing knowledge about effective decision-makings in CLSCs.

In the first part of the thesis, we study joint control of stochastic forward and stochastic reverse material flows in CLSCs. With an application to a CLSC where postconsumer products are collected for warranty service purposes, we demonstrate that the benefit of coordinating two production activities could be significant. We develop a model that can be used to obtain an effective inventory control policy for coordinating forward and reverse material flows. Through Monte Carlo simulation and global sensitivity analysis, we identify major influential factors that affect system's warranty cost savings performance. The results indicate that joint control of forward and reverse material flows greatly improves warranty cost savings performance as well as system's robustness to uncertainties.

The second part of the thesis develops a differential game model for characterizing decentralized time-varying competitive decision-making in a CLSC. The differential game model is particularly useful for studying time-varying interactive decision-making in CLSCs that involve many stakeholders who pursue different objectives in forward and reverse production activities. We identify optimal prices and production strategies that evolve over time under fluctuating market demand. Also, the model provides a quantitative scheme that can be used to obtain an efficient apportionment of product recovery processes.

The third part of the thesis describes the relationship among consumers' risk-aversion, product cannibalization of new products by remanufactured products, and growth of CLSCs through price optimization models. Whereas price is one of the most effective variables for managing market demand, previous CLSC research has mainly focused on operational problems without paying much attention on the interface between CLSCs and markets. We develop models that jointly determine optimal prices in forward and reverse channels considering consumers' willingness-to-pay (WTP) for remanufactured products, consumers' willingness-to-accept (WTA) for a buyback price, and consumers' risk aversion to uncertain quality perceptions. The results show that consumers' active participation in CLSC is an important factor for the viability and growth of a CLSC. Also, we show that companies can benefit from product remanufacturing although it may be accompanied by production cannibalization.

CHAPTER I

Introduction

In 2008, 3.2 million tons of electronic waste was generated in the U.S., 86.4 percent or 2.7 million tons of which was trashed in landfills or incinerators (*Electronics Take-Back Coalition*). The potential value embedded in the waste steam is significant. The amounts of gold and copper contained in one metric ton of electronic circuit boards exceed 40 and 30 times the concentrations of gold and copper ores mined in the US, respectively.[1] While valuable in recovery, these materials pose serious environmental and health risks when discarded in landfills. Electronic waste accounts for 75 percent of harmful heavy metals found in landfills (*Li et al.*, 2009).

1.1 Closed-Loop Supply Chains

Closed-Loop Supply Chains (CLSCs) that integrate the activities for reclaiming residual values in postconsumer products with the traditional forward supply chain activities are important from financial and environmental perspectives. CLSC can help companies not only improve profit gains through efficient use of energy and materials in their supply chains, but also resolve environmental and legal compliance issues under rapidly increasing societal concern and regulatory pressure for "going green." However, unlike traditional forward supply chains, in which the major source of uncertainty is fluctuating market demand, a much higher level of uncertainty is generally associated with return flows of postconsumer products, complicating the decision-making in CLSCs. For profitable and sustainable growth of CLSCs, it is

[1] http://www.epa.gov/osw/conserve/materials/ecycling/faq.htm

crucial to acquire the ability to make optimal decisions regarding design, operations, and market adaptation considering the potential influence of uncertainty in the return flows.

1.2 The Research Topics

This dissertation consists of three studies, each of which develops quantitative models for the analysis of effective decision-making in CLSCs with an emphasis on understanding the influence of uncertainty in the return flows of postconsumer products, time-varying market demand, and consumers' risk preferences. In what follows, we summarize the research topics.

Joint control of forward and reverse material flows. In Chapter II, we address the problem of joint control of forward and reverse material flows in CLSCs as a way of achieving cost savings and counteracting the impact of uncertainty in the return flows of postconsumer products. Return flows of postconsumer products are generally characterized by a high level of uncertainty in product quality and collection amount, unlike the forward material flows that can be relatively well planned and controlled. The uncertainty propagates through the system and eventually impacts the forward material flows when return flows join the forward transaction channel after undergoing a product recovery process. This interaction between forward and reverse material flows complicates the decision-making, e.g., inventory planning, in CLSCs.

As an application, we consider a closed-loop warranty service model that processes demands for new products and warranty claims with product take-back activities. We frame this as a multiperiod random yield inventory control problem that extends the open-loop warranty inventory model discussed in *Huang et al.* (2008). Whereas they solve Markov Decision Processes (MDPs) to obtain optimal inventory policies for a single inventory location that supports demand and warranty claims, the inclusion of stochastic return flows increases the model complexity in our case and makes the MDP approach analytical and computationally intractable. We instead take a simpler approach to work around this issue. That is, we approximate the multiperiod problem with a single period problem. This approximation method has been shown to produce high quality solutions (*Bollapragada and Morton*, 1999; *Inderfurth and Transchel*, 2007). This method also gives exact solutions to the models discussed in *Huang et al.* (2008).

We examine two closed-loop warranty service models, i.e., a coupled system (i.e., inventory for new products and inventory for recovered products are jointly controlled) and a decoupled system (i.e., two distinct inventories are independently controlled), and compare them to the open-loop warranty service model to quantify the benefit of product recovery in cost savings. We conduct Monte Carlo simulation and global sensitivity analysis to investigate the impact of uncertainty on system performance. The results show that closed-loop warranty service models save warranty service costs when the amount of uncertainty in the return flows is low, the unit cost savings from product recovery is high, and a target warranty service level is not excessive. Also, joint control of forward and reverse material flows significantly increases the robustness of a CLSC to uncertainty in the return flows, making the system perform always no worse than the open-loop system.

Differential game in CLSC. Chapter III examines the temporal aspect of optimal pricing and inventory decisions for decentralized CLSC decision makers who engage in both forward and reverse production activities under a time-varying environment. This work has recently been published (*Lee et al.*, 2011). Rapid advances in technology and decreasing product life cycles have resulted in massive generation of waste steams that contain enormous residual value, which one can easily observe, for example, in the consumer electronics industry. This is one clear example of how the benefit of CLSC can be maximized. Yet conflicting objectives of manufacturers and collectors make it difficult to achieve a time-varying equilibrium in CLSCs when market demand changes over time. The novelty of the proposed approach is that, instead of assuming stationary exogenous and endogenous system conditions, as is common in the literature, we employ differential dynamic game model to address the influence of nonstationary market demand on time-varying key decisions in CLSC. The objectives are (i) to determine decentralized pricing and production strategies, in forward and reverse channels, which evolve over time, and (ii) to characterize the influence of nonstationary market dynamics on the decision-making in CLSCs. From the model we obtain useful insights on (i) the characteristics of optimal decentralized dynamic pricing and production planning in forward and reverse channels, (ii) the mathematical allocation mechanism for product recovery processes among agents, and (iii) the benefit of a dynamic game model over static models for addressing decentralized decision-making in CLSCs.

Optimal prices, risk aversion, and product cannibalization. The third topic is presented in Chapter IV, where we focus on interfacing operational problems with marketing issues to obtain useful insights on optimal product prices, product cannibalization, and the financial viability and growth of CLSCs. This research is motivated by the fact that it is not just the price, but also the quality perception that consumers take into account in their decisions for purchasing a product. We develop a model that addresses consumers' quality perceptions through a parameter that represents the level of consumers' risk aversion in their purchasing decisions. This is particularly relevant for CLSCs where the quality of remanufactured products are perceived to be inferior to that of new products. This often becomes a barrier for sustainable growth of a recycling market. For instance, 40-50% of used car tires can be resold through retreading at a price 30-40% cheaper than that of new tires (*Environment*

Policy Committee, 2005). Consumers, however, are extremely risk averse to purchasing retreaded passenger car tires, although it is guaranteed that there is no quality difference between new and retreaded tires. Risk aversion can be bilateral. Consider a remanufacturer who offers a buyback reward to customers who return their used products. Postconsumer products are generally characterized by high levels of uncertainty in residual values. In this case, establishing stable and financially viable return flows will be impeded by firms' lower willingness-to-pay for uncertain residual values in used products and consumers' lower willingness-to-accept for a buyback reward for returning their used products. As such, understanding the characteristic distributions of WTP/WTA and the influence of uncertainty on risk averse decision-making is fundamental for determining optimal prices and promoting the growth of CLSC and the markets.

Whereas the assumption of uniformly distributed WTP has been widely used in previous research, the validity of this assumption is questionable given that a real WTP distribution is unlikely to follow a uniform distribution. The information on WTP distribution is important for determining optimal prices. Thus, a solution obtained from a uniformly approximated WTP distribution may directly impact companies' profit performances in real markets. We show that the uniform WTP assumption can produce a solution that is infeasible in a real market, or, if it is feasible, is of low quality.

We relate consumers' risk aversion to the problem of product cannibalization. The 'fear of product cannibalization' is widespread among many manufacturers. This is a major barrier for the growth of product and material recycling. There is, however, no thorough scientific evidence for justifying the belief on the adverse effect of product cannibalization. Through quantitative models, we examine the influence of uncertainty and risk aversion of decision makers on product cannibalization, market growth, and profit performance of a CLSC. We show that companies can pursue profit increases by diversifying their product portfolio with new and remanufactured products, although this may be accompanied by product cannibalization.

1.3 Contributions and Future Research Directions

CLSCs are complex systems that are significantly different from traditional forward supply chains. The methodologies used for understanding the latter may not be directly applicable to understanding effective design and operation of CLSCs. This dissertation develops models and analysis on three topics novel to the field of CLSC research. The approach in Chapter II is new in the sense that it combines a multi-period stochastic inventory problem, a random yield problem, and a multiple inventory transshipment problem within the context of CLSC in order to facilitate better insights and new knowledge on effective product recovery operations with stochastic return flows. The dynamic game model presented in Chapter III is the first one of its kind for CLSC analysis. In particular, the model enables useful insights for designing effective product recovery processes, the knowledge of which is not obtainable from a static game model when we need to consider time-varying market demand. The price optimization models in Chapter IV are significant as the first attempt in the CLSC research field to quantify the effect of consumers' risk aversion to remanufactured products and its implication on the system performance and product cannibalization. The models and results challenge the unjustified belief that product cannibalization is bad and promote active engagement in product remanufacturing.

The three research topics discussed in this dissertation can be extended to explore several interesting problems. These are discussed in Chapter V.

CHAPTER II

Multiple Inventory Control in Closed-Loop Supply Chains for Warranty Service

2.1 Introduction

Product warranty is critical for helping companies increase market share and customer loyalty by guaranteeing product quality and promoting customer satisfaction. Yet the benefit of warranty service comes with significant costs. In 2010, warranty claims cost U.S.-based computer manufacturers about \$5 billion.[1] Computer manufacturers try to save warranty costs, for instance, by shortening warranty period, passing the burden of warranty service back to their suppliers, and reducing failure rates in their products. Nevertheless, products fail. In 2010, product warranty liability of Hewlett-Packard, the number one warranty service provider among computer manufacturers in terms of warranty service cost, was \$2.4 billion compared to an operating profit of \$11.5 billion (*Hewlett-Packard Annual Report*, 2010). Dell spent more than \$1 billion on warranty claims in 2010. Warranty claims rates are higher for portable devices such as cell phones. Apple has seen steady rise in warranty claims and it spent \$250 million in the first quarter of 2011.

2.1.1 Closed-loop supply chain for warranty cost savings

In this chapter we investigate effective design and operation of CLSCs that utilize postconsumer products as a way to save warranty costs. Manufacturers remanu-

[1] All data and facts in this paragraph are from http://www.warrantyweek.com/archive/ww20110421.html

facturer used products for warranty service purpose. For example, Hitachi Global Storage Technologies, Inc. remanufactures defective hard drive returns for warranty service (*Khawam et al.*, 2007). This practice, however, is complicated by the existence of a high level of uncertainty in the return flows of postconsumer products. A wide variety of quality differences exists in postconsumer products, which makes it difficult for manufacturers to setup product recovery plan as the number of repairable items may vary significantly from one period to another. The situation becomes even more complicated when the uncertainty propagates through the system and eventually impacts the forward material flows. For instance, new items may have to be used to meet warranty claims when there is a shortage in repairable items, but how many will be needed depends on how many of collected postconsumer products and returned defective items will be repairable. This is a critical problem in the operation of CLSCs where the traditional forward supply chain activities need to be integrated with the reverse production activities for better productivity and profitability. Most manufacturers, however, separate the operations of the forward and reverse production processes (*Debo et al.*, 2005). A quantitative model that explains the interaction between forward and reverse material flows is crucial for assisting optimal decisions, e.g., an integrated inventory planning, in the CLSCs.

The importance of this research lies in the characterization of optimal inventory policy that jointly controls the flows of new and postconsumer products for satisfying demand and warranty claims in a cost effective manner. Our model reveals that joint control of forward and reverse material flows greatly increases the system robustness to the impact from uncertainties carried in by the return flows. For those companies that operate independent forward and reverse production systems, we characterize, through Monte Carlo simulation and global sensitivity analysis, conditions when their warranty cost saving performance can be improved by transitioning to joint control inventory policy.

2.1.2 Modeling and solution approach

We develop closed-loop warranty service models that process demand and warranty claims utilizing product take-back activities. We frame this as a multiperiod random yield inventory control problem that is built on and extends the open-loop warranty inventory model discussed in *Huang et al.* (2008). They address the problem through Markov Decision Processes (MDPs) models and report that, in the case of a manufacturer who uses new items to replace defective items, up to 69% of warranty service costs could be saved by taking into account potential warranty claims in inventory control. We show that CLSCs achieve further cost savings. As for modeling and solution approach, the MDP is analytically and computationally intractable in our case as the inclusion of stochastic return flows significantly increases the model complexity. We instead take a simpler approach to work around this issue. That is, we approximate the multiperiod problem with a single period problem. This approximation method has been shown to produce high quality solutions (*Bollapragada and Morton*, 1999; *Inderfurth and Transchel*, 2007). This method also gives exact solutions to the model discussed in *Huang et al.* (2008). We examine two closed-loop warranty service models, i.e., a coupled system (i.e., inventory for new products and inventory for recovered products are jointly controlled) and a decoupled system (i.e., two distinct inventories are independently controlled), and compare them to the open-loop warranty service model to quantify the benefit of product recovery in warranty cost savings. We conduct Monte Carlo simulation and global sensitivity analysis to investigate the impact of uncertainty on system performance. The results show that closed-loop warranty service models are effective for saving warranty service costs given that the amount of uncertainty in the return flows is low, the unit cost savings from product recovery is high, and a target warranty service level is not excessive. Also, joint control of forward and reverse material flows significantly increases the robustness of CLSC to uncertainty in the return flows, making the system perform always as well as or better than the open-loop system.

One of the major barriers to efficient warranty inventory management with product recovery is uncertainty in the return flows of postconsumer products. This is because inventory control must deal with a *random supply* of postconsumer products as well as random demand from the market, which is different from the inventory control of a traditional supply chain for forward material flows, where uncertainty is usually assumed to exist only in future demands. A firm that engages in take-back of postconsumer products generally faces a wide range of variability in the quality of used products, which is unknown to the firm at the time of collection due to different usage patterns of consumers.

Therefore, we are particularly interested in the influence of uncertainty in the return flows on warranty service cost savings. It is known that lateral transshipment between multiple inventory locations is effective for protecting systems from the impact of uncertainty in demands. Motivated by this, we explore the potential benefit of operating two parallel inventory locations for new and recovered products. Namely, there are two distinct inventories: one for new products and the other for recovered products. The first one, which we call the *forward production system*, will be mainly influenced by the uncertainty in demand for new products, which is relatively well understood by the research community. The latter, which we call the *reverse production system*, will be directly influenced by the uncertainty in the return flows. Additionally, the forward production system can supply the reverse production system when the latter suffers from a shortage in recovered products. We assume that the reverse production system cannot transship its recovered product to the forward production system. This is because, in general, new products are made of new parts. Such a structure is also in line with the recent trends, i.e., "firms are actively integrating their after-sale parts and services with their forward supply chain processes" (*Huang et al.*, 2008). As such, our model generalizes the open-loop warranty service modes (*Huang et al.*, 2008), random yield inventory model (*Bollapragada and Morton*, 1999), and two-location transshipment model (*Rudi et al.*, 2001) within the context of a closed-loop supply chain.

To illustrate the potential benefit of joint control of forward and reverse material flows, we investigate and compare three different cases: (i) an open-loop warranty service system, (ii) a decoupled closed-loop warranty service system, and (iii) a coupled closed-loop warranty service system. Case (i) is the one that is discussed in *Huang et al.* (2008) and will be used as a reference case for examining the potential benefit of our closed-loop warranty service models. In case (ii), the forward and reverse production systems are independently operated. In case (iii), the forward and reverse production systems are linked with lateral transshipment policy. Thus, the firm in case (iii) has three options for processing warranty service requests: (i) use new items to replace defective items, (ii) repair, if possible, failed items, or (iii) collect postconsumer products from the market and recover them for providing warranty service. The second and third options serve best for the purpose of saving warranty costs as they are often significantly less costly than the first option when collection costs are low enough to enable cost savings from unit product recovery.

In CLSC literature, product reclamation processes are classified as recycling, remanufacturing, refurbishment, and reuse depending on the level of dematerialization. Since our model can address all these different cases, we use the term 'recovery' to collectively indicate these different types of product reclamation processes.

2.1.3 Related literature

For a general introduction to issues in warranty service, we refer readers to *Murthy and Djamaludin* (2002) and *Murthy and Blischke* (1992). Several other research areas are related to our study. The first one is a periodic review inventory system with product recovery. *Simpson* (1978) derives a three-parameter optimal inventory policy, i.e., recover-up-to, order-up-to, and discard-down-to, for an inventory system with stochastic demands and stochastic returns, where returns can be stored or discarded before they are recovered. *Inderfurth* (1997) extend this result considering the impact of lead time on the optimality of the inventory policy. *van der Laan et al.* (1999) analyze "push" and "pull" remanufacturing policies in an inventory system with product recovery. *Fleischmann et al.* (2002) extends a standard stochastic inventory model by including stochastic returns. In their model, Poisson demands and Poisson return flows are assumed to be independent of each other. There may exist correlations between demands and returns, but the benefits of the independence assumption can outweigh the alternative of model complexity and the difficulty of accurately capturing the correlation. A similar assumption is used by *DeCroix* (2006). *Kiesmüller and Minner* (2003) discuss a joint inventory control problem for determining optimal produce-up-to and remanufacture-up-to levels through a newsvendor type modeling approach. Two inventory locations, one for new and the other for returned products, are considered, but recovered products finally join the inventory for new products and there is no differentiation between new and recovered products. As for multi-echelon systems, *DeCroix et al.* (2005) show the optimality of a stationary base-stock policy in a series inventory system where returns are modeled as negative demands. *DeCroix and Zipkin* (2005) study the impact of product returns on inventory management in an assembly system. *DeCroix* (2006) shows that when return flows enter the upper most stage in a serial multi-echelon inventory system, an optimal solution is obtained by solving a sequence of single-stage problems. This is also the case for a serial system without returns discussed by *Clark and Scarf* (1960). Common assumptions employed by the aforementioned papers are passive stochastic returns, product recovery with no yield loss, stochastic demands, and perfect substitution of recovered products for new products. The last assumption does not hold in some practical cases, especially when consumers tend to differentiate between new and recovered products, e.g., new and used cellphones, new and retreaded tires, new and used books, etc. In this case, one needs to model two distinct output streams, i.e., new and recovered from the system, which are supported by two separate inventories. We contribute to this literature by expounding the control of multiple inventories for processing multiple types of demands with product recovery.

In general, the yield from return flows and warranty returns is not perfect. This aspect is addressed by *Khawam* (2009) who studies a single-location inventory control problem for warranty service. Warranty returns are repaired at a random yield rate less than one; thus, new products are used to compensate for any shortage in warranty inventory. In this chapter, we consider three different sources of supply to the system: new materials, repaired warranty returns, and recovered postconsumer products. Among these, the last two sources exhibit uncertainty which will be modeled by random variables. This makes our model a multiperiod stochastic inventory control problem with random yield. Whereas certain classes of multiperiod problems are known to have myopic solutions with efficient computational properties, multiperiod inventory problems with unreliable supply do not generally have myopic solutions (*Arrow et al.*, 1958). Nevertheless, myopic solutions are known to provide near optimal performance for a wide range of stochastic inventory control problems (*Bollapragada and Morton*, 1999; *Inderfurth and Transchel*, 2007).

Unreliable supply introduces another difficulty in the inventory control problem, e.g., balancing demand and supply. As is noted in their review of random yield inventory problems, *Yano and Lee* (1995) point out that one of ways to mitigate the impact of unreliable supply is to allow recourse actions. In practice, inventory lateral transshipment among firms as a recourse action has proved to be effective for increasing service level and reducing costs (*Minner et al.*, 2003). *Robinson* (1990) and *Rudi et al.* (2001) study single period transshipment problems. We combine the transshipment problem and the random yield stochastic inventory control problem within a reverse supply chain context in order to investigate the benefit of the recourse action in the presence of uncertain supply sources.

Lastly, there is a stream of research that directly mentions inventory management for warranty service. *Huang et al.* (2008) show through a Markov Decision Processes (MDP) modeling approach that a base stock policy is optimal for an open-loop supply chain that processes customer demand and warranty service requests. They do not consider product repair, but new products are used to meet both types of demands. As opposed to the existing research in the field, our model differentiates new and recovered products by implementing two distinct inventory locations, one for new and the other for recovered products. These two inventories are linked by transshipment policy from the forward production system to the reverse production system. Our study extends the model for an open-loop supply chain for warranty service discussed in *Huang et al.* (2008).

The rest of this chapter is organized as follows. In section 2.2 we briefly describe the open-loop supply chain warranty service model with an alternative solution procedure to the MDP approach used in *Huang et al.* (2008). In section 2.3 we develop the closed-loop supply chain warranty service model and its solution. In section 2.4 we discuss the characteristics of the solution to the closed-loop model. In the final section, we summarize the results with concluding remarks.

2.2 Open-Loop Warranty Service Model

Huang et al. (2008) study an optimal inventory policy in an open-loop supply chain that faces demands for new items and requests for warranty services as depicted in Figure 2.1. Both types of demands are supported by one single inventory for new products which are supplied by an external supplier. The authors obtain optimal solutions to discounted finite horizon, discounted infinite horizon, and average cost infinite horizon models using Markov Decision Processes. We show that there is an

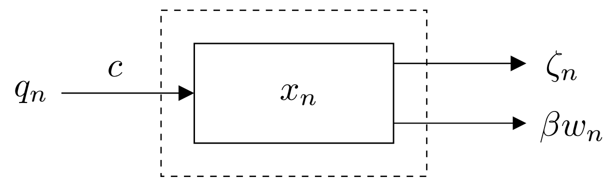


Figure 2.1: Base case: open-loop warranty service model. The problem is to determine the ordering quantity q_n when the initial inventory is x_n in order to satisfy both of demands ζ_n and warranty service requests βw_n at a minimum cost.

alternative method that gives the same solutions. This method provides a simpler solution procedure for more complex models that are discussed in §2.3.

First we briefly describe the open-loop warranty service model and the optimal inventory policy within the context of a multiperiod newsvendor problem. Consider a firm that faces random demand ζ_n and requests for warranty service βw_n, where β denotes the failure rate and w_n is the total number of products being covered by the warranty policy in period n. A detailed expression for w_n is provided later in this section. The product warranty is assumed to be renewable, i.e., once a product is repaired through the warranty service, the warranty period for the product is renewed. The firm's problem is to determine the optimal order up to level y_n or ordering quantity q_n in each period n, given an initial inventory level x_n, i.e., $y_n = x_n + q_n$. The unit ordering cost is c. Any leftover item can be returned at the original purchase price c, and any backlog at the end of each period can be fulfilled at the same purchase cost c in the next period. The time value of money discount factor is denoted by α.

Depending on the actual demand realization, a unit holding cost h or shortage penalty p is assessed. These are the two main trade-offs that drive the optimization process in the model. When the system carries more inventory than is needed to meet demands, the holding cost incurred at the end of period n is simply computed by $h(y_n - \beta w_n - \zeta_n)^+$.

The computation of inventory shortage cost is more involved than that of the inventory holding cost because the product warranty policy applies to backlogged demand with a time delay. More specifically, let p_o denote the unit shortage penalty for the case when the firm sells products *without* warranty. Let us now examine how the warranty policy alters p_o. The product warranty period comes in effect the moment a customer begins using the product. This implies that the firm does not have to consider any potential warranty claims for currently backlogged items because these items are not in use by customers yet. The warranty period for backlogged items begins only when the backlog orders are fulfilled, which happens in the following period. In short, the warranty period for normal sales begins in the current period but it is delayed by one period for backlogged items. Let $\bar{c}_w$ denote the discounted expected future warranty cost incurred by an item that is currently covered by the warranty policy. Among those currently covered by warranty policy, a fraction β of them is assumed to need warranty service in the current period. The cost of replacing each failed item by a new unit is c, i.e., the unit ordering cost. Thus, the expected cost of providing warranty service in the current period is $c\beta$ per item. In the next period, the warranty policy remains effective with a rate δ and the warranty cost is discounted by α. This gives $\alpha c\beta\delta$ as the expected warranty cost in this period. Similarly, we have $\alpha^2 c\beta\delta^2$ as the expected warranty cost in the subsequent period. We now obtain the following expression for $\bar{c}_w$

$$\bar{c}_w = c\beta + \alpha\delta c\beta + \alpha^2\delta^2 c\beta + \cdots = \frac{c\beta}{1-\alpha\delta}. \tag{2.1}$$

Therefore, the unit shortage penalty cost p has the following expression

$$p = p_o - \bar{c}_w + \alpha\bar{c}_w. \tag{2.2}$$

In other word, the unit shortage cost is adjusted by a one period discounted average warranty cost. Note that $p < p_o$ and the difference between p and p_o comes from not providing warranty service for backlogged items.

Let $C_n(x_n)$ be the minimum expected discounted cost over periods $n, n+1, \ldots, N$ beginning with an inventory x_n. We obtain the following recursion:

$$\begin{aligned} C_n(x_n, w_n) = \min_{y_n \geq x_n} \Big\{ & c(y_n - x_n) + hE\left[(y_n - \beta w_n - \zeta_n)^+\right] + pE\left[(\beta w_n + \zeta_n - y_n)^+\right] \\ & + \alpha E\left[C_{n+1}(x_{n+1}, w_{n+1})\right] \Big\} \end{aligned} \tag{2.3}$$

where

$$x_{n+1} = y_n - \zeta_n - \beta w_n \tag{2.4}$$

$$w_{n+1} = \delta\left[(1-\beta)w_n + \min\{y_n, \beta w_n + \zeta_n\} + [x_n]^-\right]. \tag{2.5}$$

The right-hand-side of (2.5) consists of

- $(1-\beta)w_n$ = the number of products that did not receive warranty service in the previous period,
- $\min\{y_n, \beta w_n + \zeta_n\}$ = the number of items delivered to the market, and
- $[x_n]^-$ = the number of backlogged items.

The assumptions for the terminal values are costless return and backlogging without penalty. Any product left over at the end of horizon can be returned at cost and any backlog can be fulfilled at the original cost. This assumption facilitates a stationary solution to the multiperiod model (*Arthur F. Veinott*, 1965). One such example of the terminal value $C_{N+1}(x_{N+1})$ is given by

$$C_{N+1}(x_{N+1}) = -cx_{N+1} + \frac{c\beta w}{1-\alpha\delta} \tag{2.6}$$

where the first term accounts for the terminal inventory cost and the second term represents the expected discounted warranty cost for w items in the last period. Let $M(x_n, w_n) = C_n(x_n, w_n) + cx_n$. The recursion (2.3) becomes

$$M_n(x_n, w_n) = \min_{y_n \geq x_n} \left\{ c(1-\alpha)y_n + hE\left[(y_n - \beta w_n - \zeta_n)^+\right] + pE\left[(\beta w_n + \zeta_n - y_n)^+\right] \right.$$
$$\left. + \alpha E\left[M_{n+1}(x_{n+1}, w_{n+1})\right] \right\} + \alpha c(\mu + \beta w_n). \tag{2.7}$$

Let us consider a myopic solution y_n^m to (2.7), i.e., a solution that minimizes

$$c(1-\alpha)y_n + hE\left[(y_n - \beta w_n - \zeta_n)^+\right] + pE\left[(\beta w_n + \zeta_n - y_n)^+\right]$$

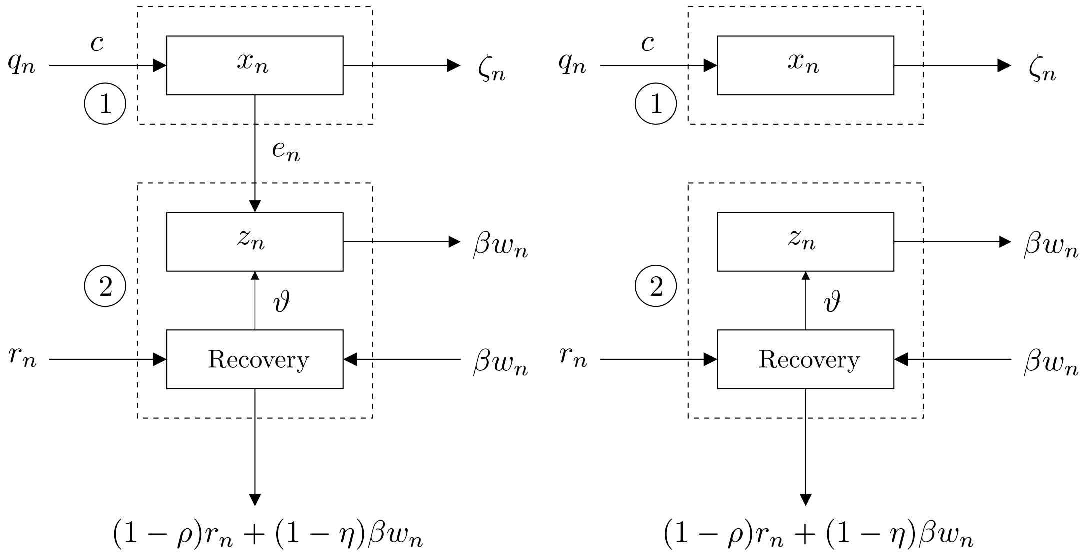
which is convex in y_n.

It is easy to show that

$$y_n^m = \beta w_n + F_n^{-1}\left(\frac{p - (1-\alpha)c}{p+h}\right) \tag{2.8}$$

for $n = 1, 2, \ldots, N$. The assumption of a stochastically increasing sequence of random variables ζ_n makes the myopic solution y_n^m an optimal solution to (2.7) (*Gerchak and Henig*, 1989; *Johnson and Thompson*, 1975; *Arthur F. Veinott*, 1965). *Huang et al.* (2008) use the same assumption, i.e., a stochastically increasing demand, to derive the myopic solution (2.8) from their MDP approach. It can also be shown that for the undiscounted infinite horizon problem, the optimal solution becomes

$$y^* = \beta w + F^{-1}\left(\frac{p}{p+h}\right). \tag{2.9}$$



(a) Coupled CLSC   (b) Decoupled CLSC

Figure 2.2: Closed-loop warranty service systems. Each system consists of (1) forward production system and (2) reverse production system.

2.3 Closed-Loop Warranty Service Systems

In this section we extend the open-loop warranty service model discussed in the previous section to closed-loop warranty service models that incorporate product take-back and repair processes for warranty service purposes. The goal is to satisfy demand and warranty claims at a minimum cost. Each closed-loop model consists of a forward production system and a reverse production system with an ordering quantity q_n and a collection quantity r_n as decision variables, respectively (Figure 2.2). In the coupled case, the forward production system supports the reverse production system in case a shortage occurs at the latter. In the decoupled case, the two distinct inventories are independently controlled without any inter-transshipment. Notations are explained in Table 2.1. A solution to the decoupled case is easily obtained from the solution for the coupled case. In what follows we construct the coupled CLSC model and develop conditions that can be used to obtain a myopic solution to this random yield transshipment multiperiod inventory problem.

Table 2.1: Notation. ('$*$' = random variables, '$\star$' = decision variables)

Notation	Meaning
c	unit purchase cost
ϑ	unit recovery cost
Δ_c	warranty cost savings per unit product recovery, $c - \vartheta$
k_M	unit shortage penalty for unmet demand
k_R	unit shortage penalty for unmet warranty service request
h_M	unit holding cost of new product
h_R	unit holding cost of recovered product
x_n	initial inventory of new products in period n
z_n	initial inventory of recovered products in period n
q_n	amount of new products ordered from the supplier
r_n	amount of used products collected from the market
e_n	amount of new products used on warranty claims
w_n	amount of products under warranty
β	warranty claims rate
δ	warranty accrual rate
α	time value of money discount factor
ζ_n	$*$ demand for new product
ρ	$*$ yield rate of collected products
η	$*$ repair rate of returned products
ξ_n	$*$ transformed random demand in reverse production system, (2.10b)
y_n	$\star$ order-up-to level in forward production system, $x_n + q_n$
u_n	$\star$ collect-up-to level in reverse production system, (2.10a)
$c_v(X)$	coefficient of variation of X
μ_X	mean of X

2.3.1 Coupled closed-loop warranty service system

As described in the previous section, the firm faces two types of demands. The first type is demand for new products and the second type is requests for warranty service. Demand for new products, denoted by ζ_n in period n, is stochastic, independent and identically distributed over time periods. A request for warranty service is generated when a product fails. The product failure rate is β. With w_n denoting the number of products covered by warranty service in period n, the number of requests for warranty service is βw_n. Similar to *Huang et al.* (2008), we use a warranty accrual rate δ, or a warranty expiration rate $1 - \delta$ to model a limited warranty period. A fraction δ of products remains to be covered by the warranty policy in the subsequent period. Essentially, new products and recovered products are used for satisfying demand and warranty service requests, respectively. However, if the amount of available recovered units is not sufficient, new products may be used for warranty service, but recovered products cannot be used for meeting demands for new products. This is why the model has two distinct product inventories—one for new and the other for recovered units. These two separate inventories are connected by the lateral transshipment policy.

Whereas the inventory control at the forward production system is a typical periodic review inventory system where the distribution of demand is influenced by the random inventory lateral transshipment, the inventory control at the reverse production system is characterized by the existence of two random supplies. One stream of random supply comes from the recovery of defective units. Not all defective units are repairable, only a fraction η of them will be successfully repaired and used for providing warranty service. This fraction is unknown until the actual repair process completes. The other stream of random supply comes from the collection of postconsumer products. The yield rate, denoted by ρ, from this stream of return flows is also unknown until the actual product recovery process takes place.

2.3.2 Representation of uncertainty in the return flows

Consider the inventory of the reverse production system. The manufacturer performs product recovery over both collected items r_n and returned items βw_n, and recovers only fractions ρ and η of them, respectively. The random yield rate η of returned products will be assumed to be less than 1, which is the reason why the manufacturer wishes to collect used products r_n to fulfill warranty service requests.

Following the method used in *Bollapragada and Morton* (1999), we transform this random supply problem with random demand into one that with random demand and deterministic supply. Let us define order-up-to level u_n and random demand ξ_n in the transformed model as follows:

$$u_n(r_n) = z_n + \mu_\rho r_n + \mu_\eta \beta w_n \tag{2.10a}$$

$$\xi_n(r_n) = \beta w_n - (\rho - \mu_\rho) r_n - (\eta - \mu_\eta)\beta w_n. \tag{2.10b}$$

Given the available initial inventory z_n and the number of products currently being covered by warranty w_n, the collection quantity r_n determines the order-up-to level for the reverse production system. The transformed demand $\xi_n(r_n)$ is a random variable which depends on random yields ρ and η and the collection amount r_n. The variability of $\xi_n(r_n)$ is computed by

$$\mathrm{Var}[\xi_n(r_n)] = r_n^2 \mathrm{Var}[\rho] + \beta^2 w_n^2 \mathrm{Var}[\eta] + 2 r_n \beta w_n \mathrm{Cov}[\rho, \eta]. \tag{2.11}$$

Note that the collection amount r_n depends on the initial inventory z_n which differs from one period to another. This makes the distribution of demand $\xi_n(r_n)$ nonstationary.

Definition II.1. We define the coefficient of variation of the transformed demand $\xi_n(r_n)$ as the level of uncertainty in the return flows of the reverse production system, i.e.,

$$c_v(\xi_n(r_n)) = \frac{\sqrt{\mathrm{Var}[\xi_n(r_n)]}}{\mathrm{E}[\xi_n(r_n)]} = \frac{\sqrt{\mathrm{Var}[\xi_n(r_n)]}}{\beta w}. \tag{2.12}$$

2.3.3 Inventory lateral transshipment

If the amount of recovered product is not sufficient after all uncertainties are revealed, then a transfer e_n from the inventory for new products is initiated as needed. We consider the case where warranty claims preempt demands for customer loyalty issues. We also consider a time period that is longer than any potential shipment delay along the transshipment line. This enables us to assume zero shipment delay and to avoid increasing the complexity of model. For practical implementation of the lateral transshipment policy, physical shipment from the forward to the reverse production system is not necessary, as any new replacement item can be directly shipped to the customer from the forward production system. The expression for the recourse action e_n is then given by

$$\begin{aligned} e_n(y_n, r_n) &= \min\{(\xi_n(r_n) - u_n(r_n))^+, y_n\} \\ &= \min\{(\beta w_n - \eta\beta w_n - z_n - \rho r_n)^+, y_n\}. \end{aligned} \tag{2.13}$$

In case of shortage in the available recovered products, i.e., $\xi_n(r_n) > u_n(r_n)$, the amount of new products needed to be shipped to the reverse production system is $\xi_n(r_n) - u_n(r_n)$, but this amount will be limited by the availability y_n at the forward production system.

The inventory balance at each location is given as follows:

$$x_{n+1} = y_n - e_n(y_n, r_n) - \zeta_n \tag{2.14}$$

$$z_{n+1} = u_n(r_n) + e_n(y_n, r_n) - \xi_n(r_n) \tag{2.15}$$

For simplicity we will suppress the dependency of $u_n(r_n)$, $\xi_n(r_n)$, and $e_n(y_n, r_n)$ and use u_n, ξ_n, and e_n unless they are needed for clarity. Assuming renewable warranty, we have an expression for the number of products that are covered by warranty in the $(n+1)$st period:

$$w_{n+1} = \delta\Big((1-\beta)w_n + \min\{y_n + u_n, y_n - e_n + \xi_n, \zeta_n + \xi_n\} + x_n^- + z_n^- - r_n\Big) \quad (2.16)$$

The first term, $(1-\beta)w_n$, of the right hand side of (2.16) is the number of products that did not request warranty service in period n. The second term represents the system outputs in period n, i.e., the amount sold and serviced, which is determined by the available inventory and demands. Four possible cases are considered as follows.

- When both locations suffer from shortage, the system has to use all units that are available, i.e., $y_n + u_n$.
- Any shortage at the second location implies the shortage at the first location as well. Hence, this is equivalent to the previous case.
- When only the first location suffers from shortage, output flow from the system is $y_n - e_n + \xi_n$.
- When there is no shortage at both locations, all demands and requests for warranty service, i.e., $\zeta_n + \xi_n$, are satisfied.

We consider full backlogging. Any unmet demand or unmet request for warranty service is backlogged as is represented by $x_n^- + z_n^-$. Finally, the warranty policy expires for collected postconsumer products, r_n.

2.3.4 The costs

Ordering cost is c per unit and product recovery cost is ϑ per unit for both of failed returns and collected postconsumer products. We assume $c - \vartheta > 0$; otherwise, product recovery will not be profitable for providing warranty service. A request for warranty service is processed by a recovered product when $\xi_n - u_n < 0$, or by a new product when $\xi_n - u_n \geq 0$. Thus, the average cost, denoted by s_n, for processing a request for warranty service is computed by

$$\begin{aligned} s_n &= \vartheta Pr\left(\xi_n - u_n < 0\right) + cPr\left(\xi_n - u_n \geq 0\right) \\ &= \vartheta + \left(c - \vartheta\right) Pr\left(\xi_n - u_n \geq 0\right). \end{aligned} \tag{2.17}$$

Note that s_n is a function of r_n, that is, the collecting effort influences the average unit warranty service cost. The warranty policy begins in the next period for any unmet demand or warranty service request. This implies that there are savings from not applying the warranty policy to unmet demand and warranty service requests in the current period. Thus, the unit shortage penalty needs to be adjusted as follows.

$$\begin{aligned} \overline{k}_{i,n} &= k_i - \frac{\beta\delta s_n}{1-\alpha\beta} + \alpha\frac{\beta\delta s_n}{1-\alpha\beta} \\ &= k_i - \frac{\left(1-\alpha\right)\beta\delta s_n}{1-\alpha\beta} \quad \text{for } i = M, R. \end{aligned} \tag{2.18}$$

The shortage penalty costs $\overline{k}_{M,n}$ and $\overline{k}_{R,n}$ are dependent on collection amount r_n and this makes the analysis complicated. However, the solution procedure is greatly simplified if we consider the undiscounted case, i.e., $\alpha = 1$, in which $\overline{k}_{M,n}$ and $\overline{k}_{R,n}$ become constants k_M and k_R, respectively.

The single period overage and shortage costs at the forward production system are given by

$$C_{M,n}(y_n, r_n) = h_M E[(y_n - e_n - \zeta_n)^+] + k_M E[(\zeta_n + e_n - y_n)^+] \tag{2.19}$$

and at the reverse production system

$$C_{R,n}(y_n, r_n) = h_R E[(u_n + e_n - \xi_n)^+] + k_R E[(\xi_n - e_n - u_n)^+]. \tag{2.20}$$

Define

$$C_n(y_n, r_n) = C_{M,n}(y_n, r_n) + C_{R,n}(y_n, r_n). \tag{2.21}$$

Let $g_n(x_n, z_n, w_n)$ be the minimum expected cost over periods $n, n+1, \ldots, N$ beginning with initial states x_n, z_n, and w_n. We obtain the following recursion:

$$\begin{aligned} g_n(x_n, z_n, w_n) + G_n(x_n, z_n, w_n) = \min_{y_n \geq x_n,\, r_n \geq 0} \big\{ & c(y_n - x_n) + \vartheta(u_n(r_n) - z_n) \\ & + C_n(y_n, r_n) \\ & + E\left[g_{n+1}(x_{n+1}, z_{n+1}, w_{n+1})\right] \big\} \end{aligned} \tag{2.22}$$

where G_n is the expected cost in period n (*Bollapragada and Morton*, 1999).

Define $M(x_n, z_n, w_n) = g_n(x_n, z_n, w_n) + cx_n + \vartheta z_n$ and $\Delta_c = c - \vartheta$. Rewrite the recursion (2.22) as

$$\begin{aligned} M(x_n, z_n, w_n) + G_n(x_n, z_n, w_n) = \min_{y_n \geq x_n,\, r_n \geq 0} \big\{ & \Delta_c E[e(y_n, r_n)] + C_n(y_n, r_n) \\ & + E\left[M_{n+1}(x_{n+1}, z_{n+1}, w_{n+1})\right] \big\} + c\mu_\zeta + \vartheta \mu_\xi. \end{aligned} \tag{2.23}$$

In what follows we will consider a myopic solution (y_n^m, r_n^m) to (2.23), i.e., a solution that minimizes the single period cost, denoted by $L_n(y_n, r_n)$,

$$\begin{aligned} L_n(y_n, r_n) &= c\mu_\zeta + \vartheta\mu_\xi + \Delta_c E[e(y_n, r_n)] + C_n(y_n, r_n) \\ &= c\mu_\zeta + \vartheta\mu_\xi + \Delta_c E[e] \\ &\quad + h_M E[(y_n - e_n - \zeta_n)^+] + k_M E[(\zeta_n + e_n - y_n)^+] \\ &\quad + h_R E[(u_n + e_n - \xi_n)^+] + k_R E[(\xi_n - e_n - u_n)^+]. \end{aligned} \tag{2.24}$$

Proposition II.1. *The optimal solution* (y^*, r^*) *that minimizes (2.24) is obtained by simultaneously solving the following two equations*

$$Pr\,(\zeta < y) = \frac{k_M}{h_M + k_M} + \frac{k_R - \Delta_c - k_M}{h_M + k_M} Pr\,(y < \gamma) + Pr\,(y - \gamma < \zeta < y) \quad (2.25)$$

and

$$\begin{aligned} E[\rho \mid \gamma < 0] Pr\,(\gamma < 0) = & \frac{k_R}{h_R + k_R} E[\rho] \\ & - \frac{k_R - \Delta_c}{h_R + k_R} E\,[\rho \mid 0 < \gamma < y]\, Pr\,(0 < \gamma < y) \\ & - \frac{h_M}{h_R + k_R} E\left[\rho \mid 0 < \gamma < y - \zeta\right] Pr\,(0 < \gamma < y - \zeta) \\ & + \frac{k_M}{h_R + k_R} E\left[\rho \mid (y - \zeta)^+ < \gamma < y\right] Pr\left((y - \zeta)^+ < \gamma < y\right). \end{aligned} \quad (2.26)$$

Proof. See the appendix of this chapter for details of the proof. □

Definition II.2. The *service level* at the forward production system is defined as $k_M/(k_M + h_M)$. The *warranty service level* at the reverse production system is defined as $k_R/(k_R + h_R)$.

In traditional newsvendor problems, the critical ratio $k/(k+h)$, or the service level, determines the optimal order quantity that satisfies all demands with a probability $k/(k+h)$. In our model, two critical ratios $k_M/(k_M + h_M)$ and $k_R/(k_R + h_R)$ are embedded in (2.25) and (2.26), but the usual meaning of the critical ratio does not apply in this case due to the complex interaction of the forward and the reverse material flows.[2] Nevertheless, the equations (2.25) and (2.26) are generalized newsvendor solutions, where $k_M/(k_M + h_M)$ and $k_R/(k_R + h_R)$ play key roles for determining

[2]For example, a typical newsvendor solution has a form Prob$(D < Q) = k/(k+h)$, which can be interpreted as "an order quantity Q satisfies demand quantity D with a probability $k/(k+h)$" or "$100k/(k+h)$% of demand quantity D is satisfied with an order quantity Q." In (2.25) and (2.26), the relationship between an order quantity and a critical ratio does not have such a simple representation.

optimal order and collection quantities, respectively.

2.4 Global Sensitivity Analysis

In this section, we address the following two main questions: (1) How much warranty service costs can be saved in the closed-loop warranty service systems? and (2) What is the influence of uncertainties on the systems? In the first question, we are interested in the amount of savings in warranty service costs. Closed-loop systems utilize recovered postconsumer products which are cheaper than new products. Thus, one might conclude that both of the two closed-loop systems would lead to cost savings. However, the impact of uncertainty in the return flows, especially the uncertainty in the yield rate from collected material, could have an adverse effect on warranty service cost savings. This will be addressed by the second question. Our approach is a large-scale numerical experimentation that employs Monte Carlo simulation and global sensitivity analysis.

2.4.1 Performance measure

We need to define a measure by which the performances of closed-loop systems can be evaluated. To compute the cost that is contributed by providing warranty service, let us consider a pure forward production system, e.g., the forward production system in the decoupled CLSC model, which is a typical newsvendor system. Let J'_O denote the total cost of this system. Next, let J_O denote cost of the open-loop warranty service system (Figure 2.1). The difference between J_O and J'_O represents the cost due to warranty service. We assume that $J_O > J'_O$ as providing warranty service will only increase the total cost. Let $W_O = J_O - J'_O$ denote the cost of providing warranty service in the open-loop system. Similarly, let $J_{coupled}$ and $J_{decoupled}$ denote the total cost of the coupled system (Figure 2.2(a)) and decoupled system (Figure 2.2(b)), respectively. Then the costs due to warranty service are expressed by $W_{coupled} = J_{coupled} - J'_O$ and $W_{decoupled} = J_{decoupled} - J'_O$ for coupled and decoupled closed-loop models, respectively. We can now compare the relative performance, denoted by Y, of two closed-loop systems to the open-loop system by the following simple expressions:

$$Y_{coupled} = 100 \times (W_{coupled} - W_O)/W_O \tag{2.27a}$$

$$Y_{decoupled} = 100 \times (W_{decoupled} - W_O)/W_O. \tag{2.27b}$$

The cost of the open-loop system is used as a reference value to which the costs of the other two closed-loop systems are compared. For example, a negative value for Y implies warranty service cost savings in the CLSC model, which is desirable.

2.4.2 Factors

We choose nine factors from the cost function (2.24) as potentially influential factors and investigate their influence on system performance. The parameter values are adopted from industry practices and related literature in warranty service problems as shown in Table 2.2. We consider the range $[0.02, 0.10]$ as the rate of warranty returns. Warranty claims rates generally do not exceed 5% in consumer electronic and automotive industries.[3] In some cases, product failure rate goes beyond 10%, e.g., Microsoft Xbox 360.[4]

Huang et al. (2008) identify that the unit holding cost h_M is not a significant components of the total costs. We normalize each of cost coefficients c, ϑ, k_M, h_R, and k_R in (2.24) with h_M, or equivalently, $h_M = 1$. While this does not change the solution, we can reduce the number of factors in the sensitivity analysis. A remanufactured product is obtained at a cheaper cost, ϑ, than c for a new product. This implies $h_R \leq h_M$. Accordingly, we set the unit holding cost of a remanufactured

[3]www.warrantyweek.com
[4]www.engadget.com

product as $h_R = (\vartheta/c)h_M = (1 - \Delta_c/c)h_M$. For a given value of a holding cost h_M or h_R, we choose to specify a service level $k_M/(k_M + h_M)$ or $k_R/(k_R + h_R)$, rather than a shortage penalty cost k_M or k_R, which is generally more informative for inventory decision-making in a newsvendor approach than individual values for holding/shortage costs. Another reason for this approach is that a dimensionless factor is desirable for generalizing the interpretation of subsequent sensitivity analysis. We set the value of unit ordering cost c between 8 and 20. *Huang et al.* (2008) consider a range $[0.01, 0.3]$ for a value of the holding cost h_M when $c = 2$. With $h_M = 1$, this is equivalent to taking a value of c from $[7, 200]$. We reduce this range to $[7, 20]$ because too much variations in the unit ordering cost c can dominate the influence of other factors in the output variation. It is known that remanufacturing can save up to 70% in the case of printer cartridges.[5] Thus, we consider $[0.1, 0.7]$ for the values of Δ_c/c. This range can explain cost savings from remanufacturing for many other products such as vehicle engines[6] and power tools[7], to name a few. The values of service levels, coefficients of variation, and mean repair rates are adopted from the case study of the warranty service inventory problem for Hitachi GST addressed in *Khawam et al.* (2007).

Demand sample data are drawn from a normal distribution $N(1000, \sigma_\zeta^2)$. A different choice of a mean demand level does not distort the numerical analysis. We only need a sufficiently large number to avoid the generation of negative numbers. The random yield ρ of collected products and the random repair rate η of warranty returns are drawn from truncated normal distributions with a common support $[0, 1]$. Without loss of generality, the mean value of random yield ρ is set to 1. This is because any change in the mean value of ρ will be scaled by the decision variable r, i.e., $\rho r = (\rho/\mu_\rho)(\mu_\rho r)$ and $E[\rho/\mu_\rho] = 1$. Note that the parameter ρ appears in (2.24)

[5] http://www.inkguides.com/remanufactured-cartridges.asp

[6] http://nationalpowersupply.com/dieselengines/enginesnewsletter.html

[7] http://www.toologics.com/uncategorized/remanufactured-reconditioned-and-refurbished-power-tools/

Table 2.2: Parameter values for the numerical experiments. Nine factors are examined in global sensitivity analysis. (Numerical values are adopted from the referred sources with slight modifications.)

Factors	Min	Max	Reference
warranty claims rate, β	0.2	0.1	www.warrantyweek.com
unit ordering cost (c)	7	20	*Huang et al.* (2008)
fractional cost savings per unit product recovery, Δ_c/c	0.1	0.7	typical unit cost savings from remanufacturing, e.g., printer cartridges
service level at reverse production system, $k_R/(k_R + h_R)$	0.70	0.99	*Khawam et al.* (2007)
service level at forward production system, $k_M/(k_M + h_M)$	0.70	0.99	*Khawam et al.* (2007)
c_v of demand, $c_v(\zeta)$	0.1	0.9	*Khawam et al.* (2007)
c_v of repair rate of warranty returns, $c_v(\eta)$	0.1	0.9	*Khawam et al.* (2007)
c_v of yield rate of return flows, $c_v(\rho)$	0.1	0.9	*Khawam et al.* (2007)
mean repair rate of warranty returns, μ_η	0.2	0.7	*Khawam et al.* (2007)

only through the term ρr. The same assumption is used in *Bollapragada and Morton* (1999).

2.4.3 Sensitivity indices

To identify influential factors, we use Monte Carlo simulation, i.e., we randomly sample the value of each of nine factors from the predefined range, as shown in Table 2.2, and analyze the output variance of Y. In other word, the performance measure Y is regarded as a stochastic process whose variance is determined by the collective variations of nine factors. This method is called global sensitivity analysis (*Sobol*, 2001; *Saltelli et al.*, 2008). More specifically, we compute the following two sensitivity measures,

$$S_i = \frac{V_{X_i}[E_{X_{\sim i}}[Y|X_i]]}{V[Y]} \tag{2.28}$$

and

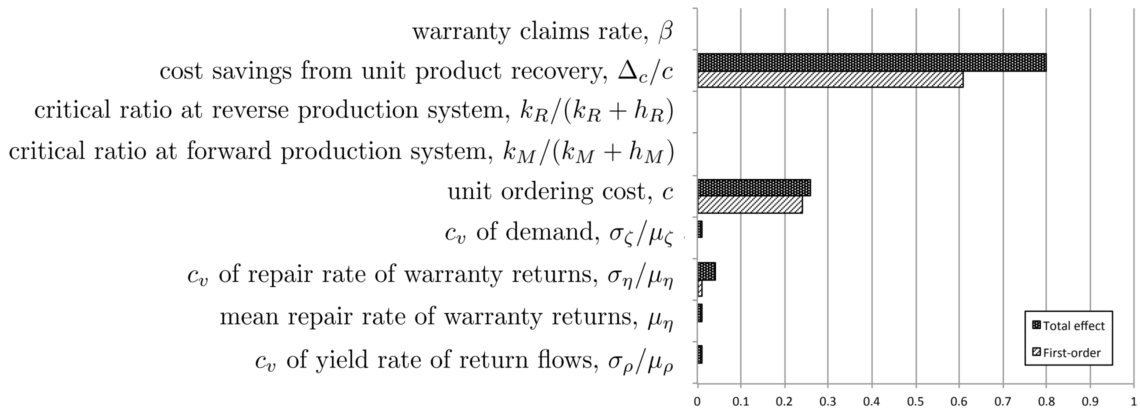
$$S_{T_i} = \frac{E[V[Y|X_{\sim i}]]}{V[Y]}. \tag{2.29}$$

The first measure (2.28) is called the *first-order sensitivity index* of a factor X_i on the output Y. The notation $X_{\sim i}$ means that the expectation is taken over all factors except X_i. The expected value $E_{X_{\sim i}}[Y|X_i]$ represents the level of output Y when the value of a given factor X_i is fixed at a specific value. The expectation is taken over all possible values of the factor X_i. We then compute the variance of these expected value, i.e., $V_{X_i}[E_{X_{\sim i}}[Y|X_i]]$. Therefore, S_i represents the fractional portion of output variance of Y, which is due to the factor X_i. A higher value of S_i implies that X_i has a more influential impact on Y. It also follows that the value of S_i is always between 0 and 1, and the sum of all first-order effects cannot be greater than 1, i.e., $\sum_i S_i \leq 1$. The second measure (2.29) is called the *total effect*. This index accounts for all higher order interactions of X_i with other factors including the first-order effect. The two measures, S_i and S_{T_i}, are referred to as Sobol indices. *Saltelli et al.* (2008) show that the first-order index is related to the standardized coefficient of a linear regression model, but the former is applicable to more general cases such as a nonlinear model while the latter is appropriate for a sensitivity analysis of linear models. Both of the methods ignore each factor's scale of units. The system output Y, as defined in (2.27), is an outcome of highly nonlinear interactions among the system parameters. Thus, (2.28) and (2.29) best serve for the sensitivity analysis of our model.

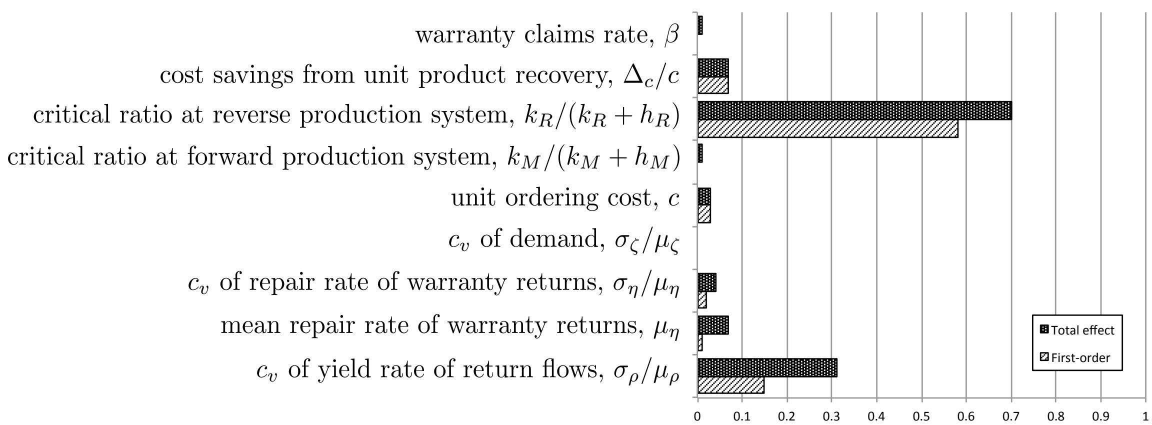
In order to compute the Sobol indices, we need to populate random samples that will be used for computing system output Y. Using the method proposed in *Saltelli* (2002), we generate two sets of 20,000 random samples from the ranges defined in Table 2.2 to estimate the Sobol indices. The estimated first-order and total effects are shown in Figure 2.3.

2.4.4 Influential factors

Let us first consider the sensitivity indices for coupled closed-loop system (Figure 2.3(a)). One can easily observe that the output variance $Y_{coupled}$ is mostly dominated by Δ_c/c. The unit cost savings per remanufacturing, Δ_c/c, is the most influential factor with a first-order index 0.61. This is an intuitive result because a higher Δ_c/c will result in higher cost savings, i.e., a decrease in the value of $Y_{coupled}$. The next most influential factor is the unit ordering cost c with a first-order index 0.24. The coupled closed-loop system receives transshipment from the forward production system. Thus, an increase in c will lead to a decrease in warranty cost savings. This will be verified in the next subsection. The variation in the mean repair rate appears to be the third most influential factor, but the fractional portion in the output variance is negligible. This implies that the contribution to the output variance from uncertainty in the return flows or in demand is virtually non-existent in the coupled closed-loop system.



(a) Sobol indices: dependent variable = $Y_{coupled}$



(b) Sobol indices: dependent variable = $Y_{decoupled}$

Figure 2.3: Estimated Sobol sensitivity indices.

The number of influential factors increases in the decoupled closed-loop system (Figure 2.3(b)) because the impact of the uncertainty in the return flow on the reverse production system is not buffered by the inventory transshipment from the forward production system. The most influential factor for this case is the critical ratio, $k_R/(k_R+h_R)$, at the reverse production system, which determines the warranty service level. This result is similar to that of *Khawam et al.* (2007) who study a single-location inventory problem that is dedicated to warranty returns. In their model, the firm initially replenishes the inventory with new products. For each of the warranty returns, a replacement item (new or remanufactured) or a monetary credit is offered. Their result shows that the system performance is significantly influenced by the service level. This is consistent with our sensitivity analysis, which indicates that the service level is the most influential factor.

In terms of first-order effect, the next most influential factor is the cost savings per unit product recovery, Δ_c/c, which is understandable for the same reason as discussed in the case of the coupled closed-loop system. However, if we look at the total effect index, the second most influential factor is the variation in the yield rate from collected postconsumer products, σ_ρ/μ_ρ. This implies that the uncertainty in the return flows of postconsumer products can significantly disrupt the system performance through interacting with other factors, unlike the case of coupled system. This is because the impact from the uncertainty in the return flow is not buffered by the forward production system through the inventory transshipment. The impact from the variation in the repair rate of defective items, σ_η/μ_η, on the output variance is also substantial.

Overall, we can conclude that the decoupled closed-loop system is highly influenced by the warranty service level and uncertainty in the return flows. This result clearly demonstrates the benefit of jointly controlling the two separate inventories with a lateral transshipment policy. By linking the reverse production system to the

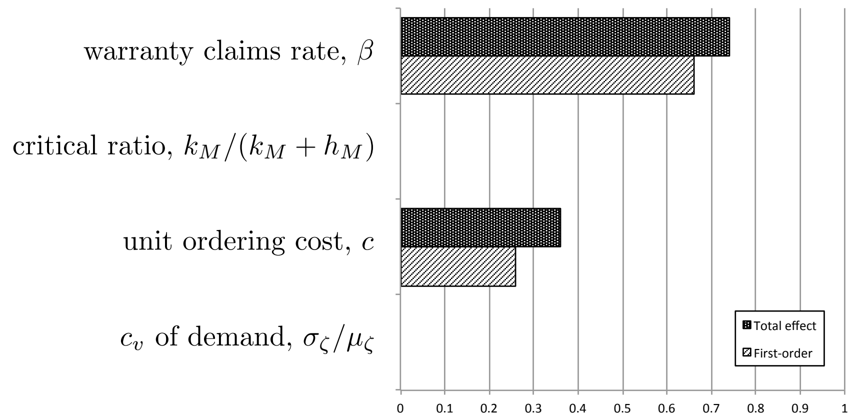


Figure 2.4: Sensitivity of warranty cost W_O to system parameters (open-loop system).

forward production system, the system greatly enhances its robustness to the impact from the uncertainty in the return flows. In the coupled closed-loop system, not only the influences of uncertainties in return flows, but also that of demand, are non-influential. This is a well-known property of the robustness of a newsvendor solution to demand uncertainty.

Further, the influence of the warranty service level becomes virtually non-existence in the coupled closed-loop system, which is important because many firms pursue a high level of warranty service for maintaining customer loyalty and brand image. The first-order effect of c is zero in the decoupled system because there is no transshipment from the forward production system.

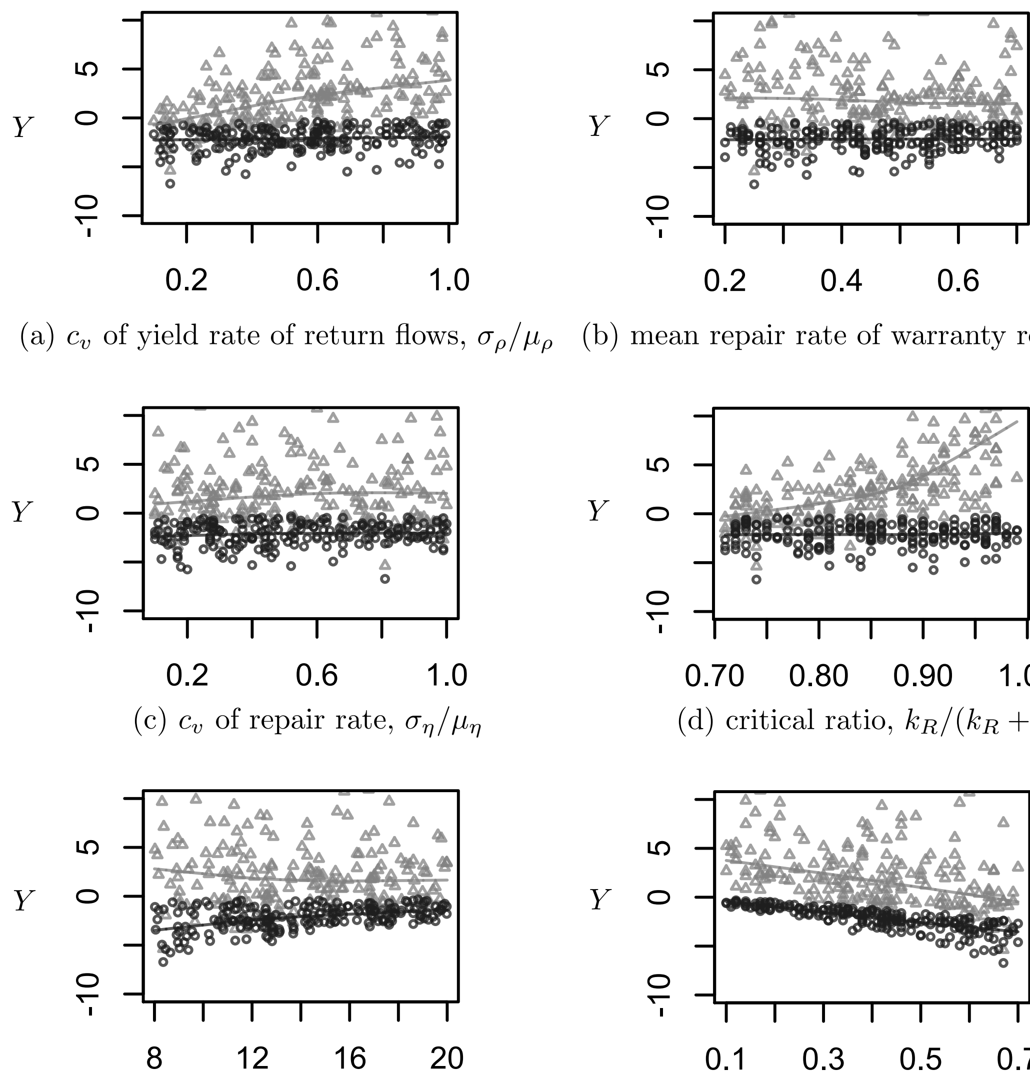
We have thus far examined the sensitivity of the *relative* performance of two CLSCs with respect to the open-loop system. This implies that a parameter, say x, that appears to be non-influential could be influential if the open-loop system is sensitive to x. In order to verify this, we show the sensitivity of warranty cost W_O in the open-loop system to various factors in Figure 2.4. There exist two influential factors, the warranty claims rate, β, and the unit ordering cost, c, in the open-loop system. The *relative* warranty cost savings performances of the CLSCs are insensitive to β because the unit cost savings Δ_c from remanufacturing is independent of the level of β, but the actual warranty cost of the open-loop system is highly sensitive to β. Therefore, β is an influential factor for the actual warranty costs of the CLSCs. On the other hand, the unit ordering cost, c, is influential in the open-loop system as well as in the CLSCs.

While the Sobol indices give information for ranking factors in terms of relative influential impact on the output variable Y, these are not informative for examining the actual change in the performance measure, i.e., increase or decrease in Y. The latter is discussed through scatter plots in the next subsection.

2.4.5 Warranty cost savings performances in the CLSCs

Figure 2.5 shows the scatter plots of the six most influential factors that are identified from the Sobol indices in Figure 2.3. The vertical axis of each scatter plot represents the system warranty cost savings performance values, $Y_{coupled}$ for "◯" and $Y_{decoupled}$ for "△." To better understand the change in the distribution of Y, we fit a locally weighted scatterplot smoothing (LOWESS) line for each case of decoupled and coupled CLSCs. For the coupled CLSC, the LOWESS lines are almost flat except for the two factors, c (Figure 2.5(e)) and Δ_c/c (Figure 2.5(f)), as they are the two most influential factors for $Y_{coupled}$. Figure 2.5(e) shows that an increase in c results in an increase in $Y_{coupled}$, which means less warranty cost savings. This verifies our reasoning for the influence of c on $Y_{coupled}$ discussed in the previous subsection. In Figure 2.5(f), we can verify that more warranty cost savings are possible with a higher Δ_c/c.

As opposed to the coupled CLSC, the decoupled CLSC shows much higher variations in the system performance $Y_{decoupled}$. This shows that the warranty cost savings in the decoupled CLSC is sensitive to internal and external 'disturbances' such as the uncertainty in the return flows (as represented by σ_ρ/μ_ρ), the uncertainty in warranty returns (as represented by σ_η/μ_η), and the warranty service level, $k_R/(k_R + h_R)$. In



(a) c_v of yield rate of return flows, σ_ρ/μ_ρ (b) mean repair rate of warranty returns, μ_η

(c) c_v of repair rate, σ_η/μ_η (d) critical ratio, $k_R/(k_R + h_R)$

(e) unit ordering cost, c (f) cost savings from unit product recovery, Δ_c/c

Figure 2.5: Scatter plots of six influential factors on warranty cost savings, Y. In each plot, the symbols "◯" and "△" indicate $Y_{coupled}$ and $Y_{decoupled}$, respectively. A negative value of Y indicates a cost saving.

particular, Figure 2.5(d) shows that the system performance rapidly deteriorates as the warranty service level increases in the range $[0.8, 0.99]$. This confirms the results in *Khawam et al.* (2007). The nonlinearity of warranty cost savings in the decoupled CLSC observed in Figure 2.5(d) is related to the nonlinear relationship between the warranty service level $k_R/(k_R + h_R)$ and the unit shortage penalty k_R. As the warranty service level approaches 1, the shortage penalty cost approaches infinity at an increasing rate.

We can use the LOWESS lines to answer our first question, i.e., how much warranty cost savings would be possible with CLSCs? The coupled CLSC is almost free from the influence of uncertainty. The system has two major influential factors, c and Δ_c/c, but these are non-stochastic quantities. Thus, decision makers can use the LOWESS lines to estimate potential warranty cost savings in the coupled CLSC without concerning much about the level of uncertainty in the return flows.

For the decoupled CLSC, we can still use the scatter plots and LOWESS lines to estimate the cost savings. However, in this case, the influence from the uncertainty in the return flows and warranty returns is significant. Let us consider two factors, the cost savings per unit product recovery, Δ_c/c, and the critical ratio, $k_R/(k_R + h_R)$. These are the two most influential factors in the reverse production system, which can be "managed" unlike the uncertainty in return flow. Let us assume that these two factors are given as fixed parameters whose values are determined by the firm's unique characteristics of the production systems and warranty service policy. This is to focus on understanding the influence of uncertainty in return flow, i.e., the variations in yield rate of return flows, σ_ρ/μ_ρ, and repair rate of warranty returns, σ_η/μ_η, on the system performance. Rather than separately investigating the influence of each of the two uncertain quantities, we are interested in the influence of the total amount of uncertainty in the return flows. To this end, we use the coefficient of variation of the transformed demand, defined in (2.12), to plot Figure 2.6 that can be used to tell where the firm is located under a given circumstance.

The horizontal and vertical axes of Figure 2.6 represent two 'deterministic' quantities that are determined by firm's unique characteristics. Depending on the amount of uncertainty in the return flows, the firm will experience an increase or a decrease in warranty service costs. The intensity of darkness indicate the absolute value of $Y_{decoupled}$. To better illustrate whether it is a decrease or an increase in cost, we use two symbols—"◯" for cost decrease and "△" for cost increase. One can observe that the plot contains two regions—one with shapes "◯", i.e., the cost saving region, and the other with shapes "△," i.e., the cost increase region. Cost savings are more likely with a higher cost savings per unit product recovery, Δ_c/c, and a lower critical ratio, $k_R/(k_R+h_R)$. A high value of Δ_c/c would be possible, e.g., with the support of well-designed product recovery processes. But a low warranty service level is unlikely in practical cases. Yet the upper right region of Figure 2.6 shows that cost savings are still possible if the level of uncertainty is low, i.e., one can observe a small "◯" symbol at the coordinate (0.45, 0.97), which are surrounded by large "△" symbols. One good example for this case is the laser toner cartridge, which is known to save up to 60% of the original manufacturing cost.

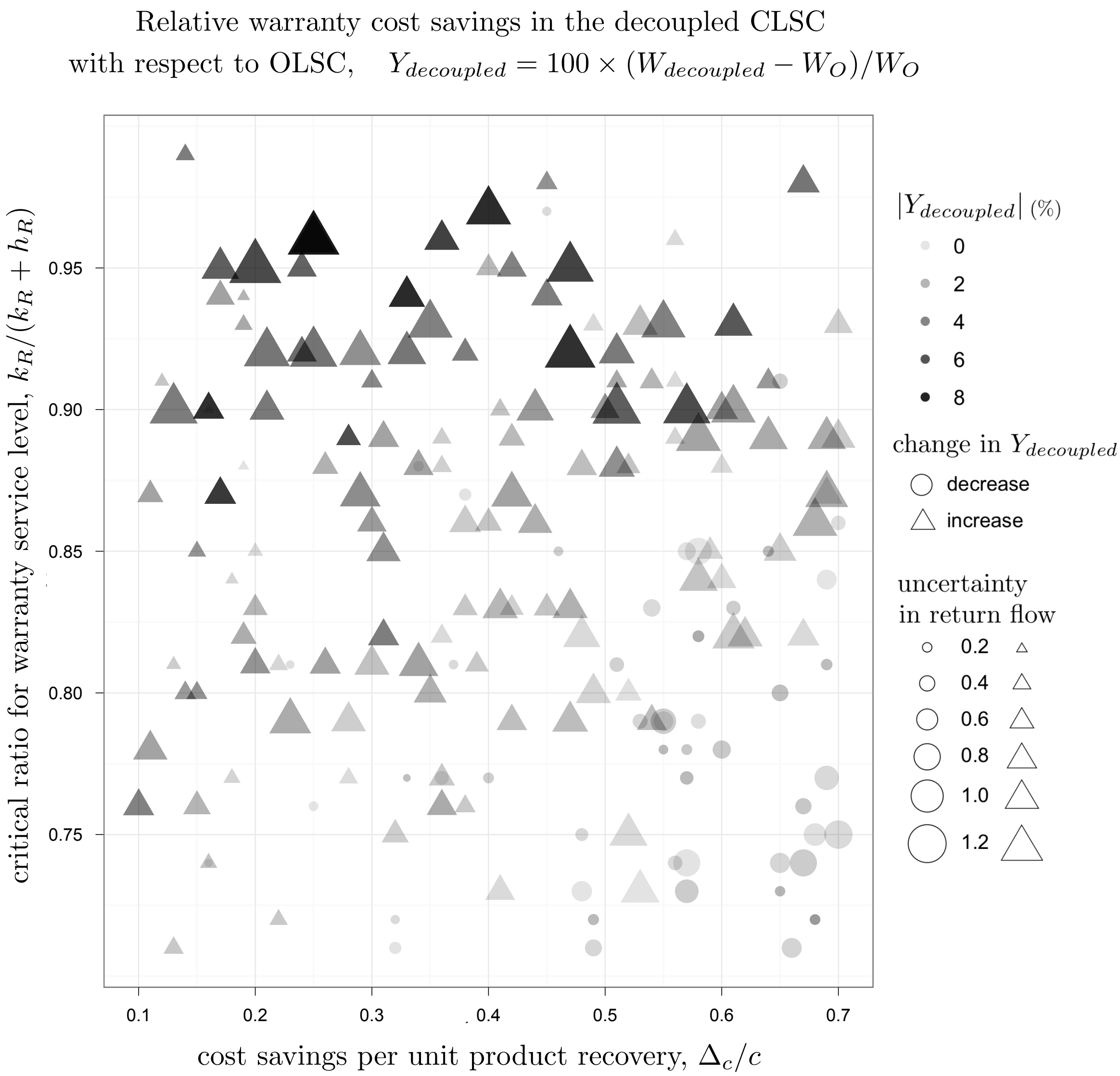


Figure 2.6: Warranty cost savings in the decoupled CLSC (200 example cases). This is a plot of the *absolute* values of $Y_{decoupled}$ as defined in (2.27b). For a given combination of warranty service level and cost savings from unit product recovery, the warranty cost savings largely depend on the amount of uncertainty in return flows. As warranty service level approaches 1, the warranty service costs in the decoupled CLSC becomes higher (i.e., darker triangles) than that in the open-loop system.

2.4.6 Benefits of the coupled CLSC

The result from the previous section implies that if a firm wishes to achieve a high warranty service level, it is desirable to jointly control the forward and the reverse material flows in its system because the coupled CLSC always performs better than the open-loop system, even when the warranty service level is close to 1 (Figure 2.5(d)). If the return flows do not save on the warranty cost, the optimization process will shut down the return flows. Thus, the warranty service cost in the coupled CLSC cannot be higher than that in the open-loop system. This is illustrated in Figure 2.7, where there are only one kind of shapes "◯", unlike the case of the decoupled CLSC. The benefits of the coupled CLSC over the decoupled CLSC and the open-loop system are summarized as follows:

- The coupled CLSC always performs better than the open-loop system.
- The coupled CLSC is robust to the impact from the uncertainty in the return flows.
- The coupled CLSC's warranty service cost savings performance is insensitive to the warranty service level.

Figure 2.8 shows that the average improvement in the warranty cost savings in the coupled CLSC over the open-loop system is almost linear to the fractional unit cost savings from product recovery Δ_c/c.

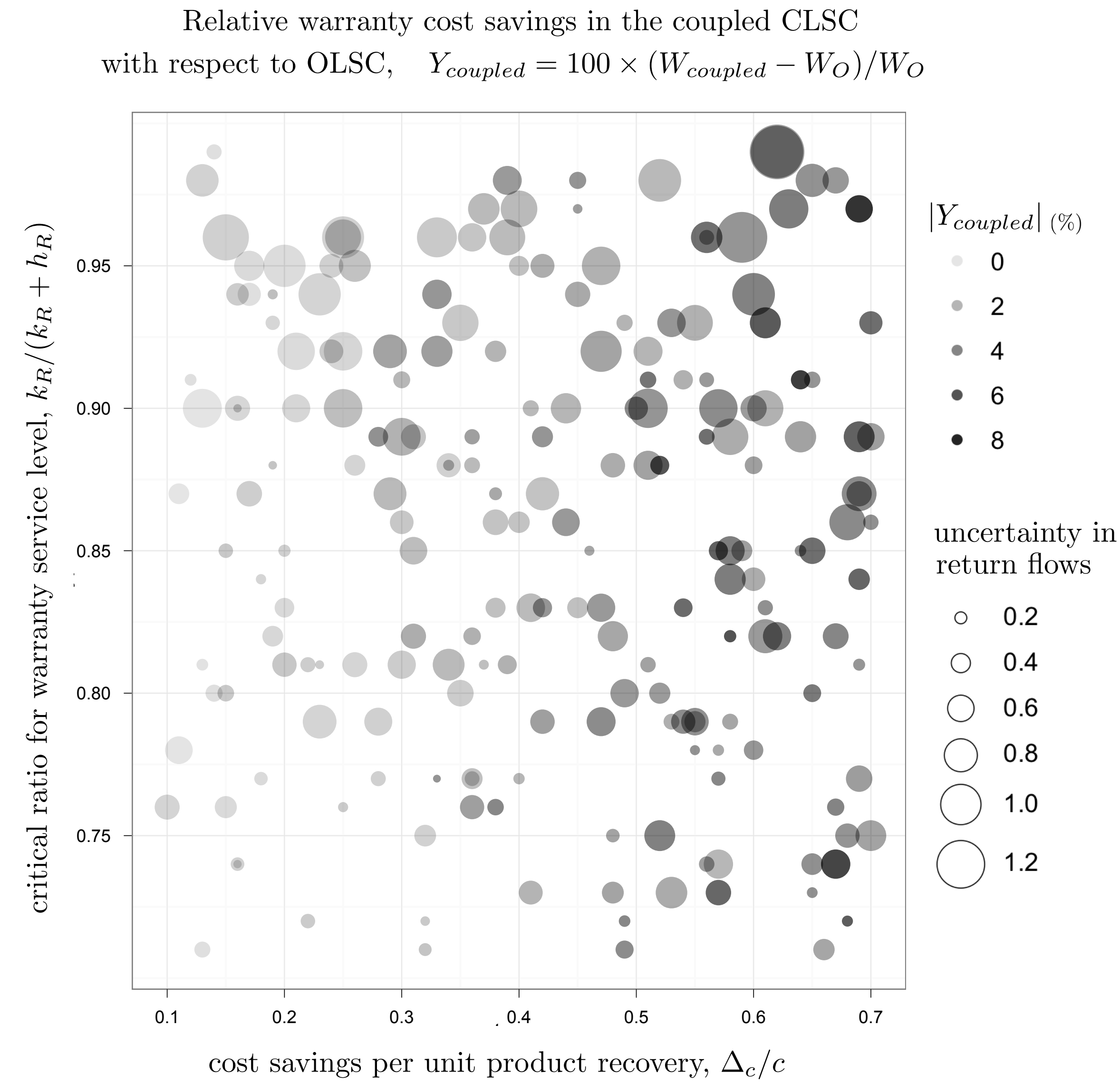


Figure 2.7: Warranty cost savings in the coupled CLSC (200 example cases). This is a plot of the *absolute* values of $Y_{coupled}$ as defined in (2.27a). The decoupled CLSC always performs better than the open-loop system. The cost savings per unit product recovery is the major influential factor that affects the warranty cost savings in the coupled CLSC. The system performance is insensitive to the amount of uncertainty in the return flows.

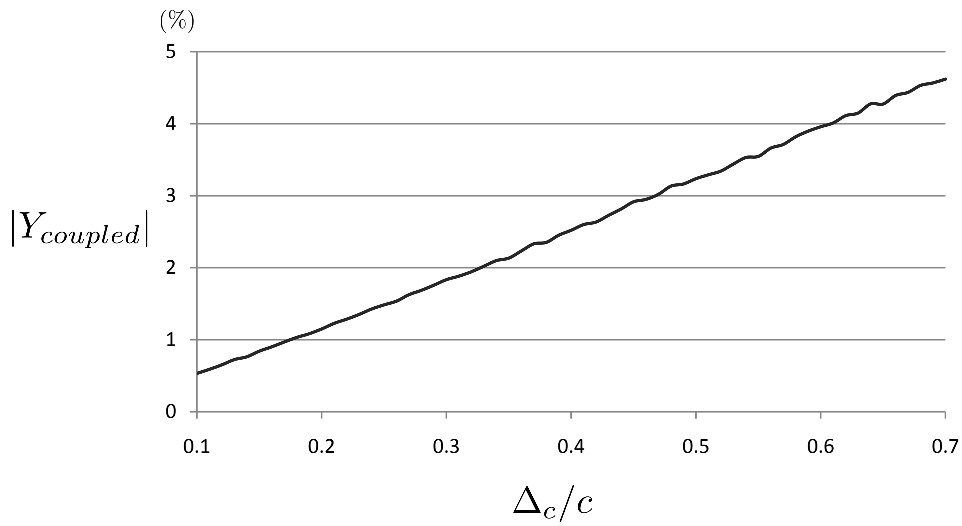


Figure 2.8: Benefit of using return flows for saving warranty costs in the coupled CLSC.

2.5 Conclusions

In this chapter we address the problem of saving warranty costs through CLSCs that collect and recover postconsumer products for processing warranty claims. In the CLSCs, new products, recovered postconsumer products, and repaired defective warranty returns are available for providing warranty services. The latter two options are generally less costly than using new products, but the overall warranty cost savings depend on how much uncertainties are embedded in the return flows and how high the firm sets the target warranty service level. We investigate two cases: coupled CLSC and decoupled CLSC. In the coupled CLSC case, forward and reverse material flows are optimally coordinated through lateral inventory transshipment from the forward production system to the reverse production system. This work develops a new model for CLSCs that integrate a random yield supply problem, an inventory transshipment problem, and a multiple-inventory control problem to address joint control of the forward and reverse material flows. We solve the models using a single period approximation and obtain myopic solutions. This approach is known to provide a high quality solution. In the case of deterministic supply, the approach can produce an exact solution with suitable assumptions.

We show that the coupled CLSC always achieves warranty cost savings while greatly enhancing the system robustness to the impact from the uncertainty in the return flows. The decoupled CLSC, where forward and reverse material flows are separately controlled, does not necessarily save warranty costs. When the amount of uncertainty in return flows and target warranty service level are high, the decoupled CLSC may perform worse than the open-loop system, i.e., the system where only new products are used to provide warranty services. We identify conditions on influential system parameters under which the decoupled CLSC performs better or worse than the open-loop system.

On the contrary, the coupled CLSC performs always as well as or better than the open-loop system. The system is insensitive to the impact from the uncertainty. There is little change in the warranty service cost in the coupled system when the warranty service level increases. The result implies that by using a uniform WTP approximation in these situations, companies that operate independent forward and reverse production systems can achieve warranty cost savings and enhance the system robustness to uncertainty by coordinating the separate material flows.

2.6 Appendix

2.6.1 Proof of Proposition II.1

For notational simplicity, we drop the time subscript n. The convexity of (2.24) follows from *Robinson* (1990), *Rudi et al.* (2001), and *Huh and Nagarajan* (2010) and we find the values of y and r that simultaneously satisfy $\partial L(y,r)/\partial y = 0$ and $\partial L(y,r)/\partial r = 0$. The following lemma will be used to derive the first order partial derivative for each term of $L(y,r)$.

Lemma II.1. *For an affine function $g(Q)$, $Q > 0$, with independent random coefficients X and Y, i.e., $g(Q) = X + YQ$, where X and Y are independent random variables,*

$$\frac{\partial}{\partial Q}\big(E\left[g(Q) \mid g(Q) > 0\right] P\left(g(Q) > 0\right)\big) = E\left[\partial g(Q)/\partial Q \mid g(Q) > 0\right] P\left(g(Q) > 0\right). \tag{2.30}$$

Proof. The expected value of the nonnegative random variable $\max\{g(Q), 0\}$ is computed by

$$\begin{aligned} E\left[\max\{g(Q), 0\}\right] &= \int_0^\infty P(g(Q) > z)dz \\ &= \int_0^\infty P(X + YQ > z)dz \\ &= \int_0^\infty P(Y > (z - X)/Q)dz \\ &= \int_0^\infty \int_{-\infty}^\infty \int_{(z-x)/Q}^\infty f_Y(y) f_X(x) dy dx dz. \end{aligned} \tag{2.31}$$

Differentiating with respect to Q gives,

$$\begin{aligned}
\partial E\left[\max\{g(Q),0\}\right]/\partial Q &= \int_0^\infty \int_{-\infty}^\infty \frac{\partial}{\partial Q}\left(\int_{(z-x)/Q}^\infty f_Y(y) f_X(x) dy\right) dx dz \\
&= \int_0^\infty \int_{-\infty}^\infty \left(\frac{z-x}{Q^2} f_Y\left(\frac{z-x}{Q}\right) f_X(x)\right) dx dz \\
&= \int_{x/Q}^\infty \int_{-\infty}^\infty \beta f_Y(\beta) f_X(x) dx d\beta \\
&= \int_{-\infty}^\infty \int_{x/Q}^\infty \beta f_Y(\beta) f_X(x) d\beta dx \\
&= E\left[\partial g(Q)/\partial Q | g(Q) > 0\right] P(g(Q) > 0). \qquad (2.32)
\end{aligned}$$

The result follows from the following relation

$$E\left[\max\{g(Q),0\}\right] = E\left[g(Q) \;\middle|\; g(Q) > 0\right] P\left(g(Q) > 0\right). \qquad (2.33)$$

□

We develop optimality conditions for (y^*, r^*) by performing marginal analysis on (2.24). For notational simplicity, let us define $\gamma := \xi - u = \beta w - \eta\beta w - z - \rho r$. For example, $e = \min\{(\xi - u)^+, y\} = \min\{\gamma^+, y\}$.

2.6.1.1 Marginal lateral transshipment costs

The expected cost of inventory lateral transshipment can be computed by

$$\begin{aligned}
\Delta_c E[e(y,r)] &= \Delta_c E\left[\min\{\gamma^+, y\}\right] \\
&= \Delta_c E\left[\min\{\gamma^+, y\} \mid \gamma^+ < y\right] Pr\left(\gamma^+ < y\right) \\
&\quad + \Delta_c E\left[\min\{\gamma^+, y\} \mid \gamma^+ > y\right] Pr\left(\gamma^+ > y\right) \\
&= \Delta_c E\left[\gamma^+ \mid \gamma^+ < y\right] Pr\left(\gamma^+ < y\right) + \Delta_c y Pr\left(\gamma^+ > y\right) \qquad (2.34)
\end{aligned}$$

from which the marginal costs of inventory lateral transshipment are obtained as

$$\begin{aligned}\Delta_c \partial E[e(y,r)]/\partial y &= \Delta_c Pr\left(\gamma^+ > y\right) \\ &= \Delta_c Pr\left(\gamma > y\right)\end{aligned} \tag{2.35}$$

and

$$\begin{aligned}\Delta_c \partial E[e(y,r)]/\partial r &= \Delta_c \frac{\partial}{\partial r} E\left[\gamma^+ \mid \gamma^+ < y\right] Pr\left(\gamma^+ < y\right) \\ &= -\Delta_c \frac{\partial}{\partial r} E\left[-\gamma^+ \mid y - \gamma^+ > 0\right] Pr\left(y - \gamma^+ > 0\right).\end{aligned} \tag{2.36}$$

From the additivity and linearity properties of conditional expectation,

$$\begin{aligned}&E\left[y - \gamma^+ \mid y - \gamma^+ > 0\right] Pr\left(y - \gamma^+ > 0\right) \\ &\qquad = E\left[y \mid y - \gamma^+ > 0\right] Pr\left(y - \gamma^+ > 0\right) \\ &\qquad + E\left[-\gamma^+ \mid y - \gamma^+ > 0\right] Pr\left(y - \gamma^+ > 0\right).\end{aligned} \tag{2.37}$$

Next, by the linearity of the differential operator,

$$\begin{aligned}&\frac{\partial}{\partial r} E\left[y - \gamma^+ \mid y - \gamma^+ > 0\right] Pr\left(y - \gamma^+ > 0\right) \\ &\qquad = \frac{\partial}{\partial r} E\left[y \mid y - \gamma^+ > 0\right] Pr\left(y - \gamma^+ > 0\right) \\ &\qquad + \frac{\partial}{\partial r} E\left[-\gamma^+ \mid y - \gamma^+ > 0\right] Pr\left(y - \gamma^+ > 0\right).\end{aligned} \tag{2.38}$$

But, $\frac{\partial}{\partial r} E\left[y \mid y-\gamma^{+}>0\right] Pr\left(y-\gamma^{+}>0\right)=0$. Thus, we can further simplify (2.36) as

$$
\begin{aligned}
\Delta_c \partial E[e(y,r)]/\partial r &= -\Delta_c \frac{\partial}{\partial r} E\left[y-\gamma^{+} \mid y-\gamma^{+}>0\right] Pr\left(y-\gamma^{+}>0\right) \\
&= -\Delta_c E\left[\frac{\partial}{\partial r}\left(y-\gamma^{+}\right) \,\middle|\, y-\gamma^{+}>0\right] Pr\left(y-\gamma^{+}>0\right) \\
&= -\Delta_c E\left[\frac{\partial y}{\partial r} \,\middle|\, y-\gamma^{+}>0\right] Pr\left(y-\gamma^{+}>0\right) \\
&\quad + \Delta_c E\left[\frac{\partial \gamma^{+}}{\partial r} \,\middle|\, y-\gamma^{+}>0\right] Pr\left(y-\gamma^{+}>0\right) \\
&= \Delta_c E\left[\frac{\partial \gamma^{+}}{\partial r} \,\middle|\, y-\gamma^{+}>0\right] Pr\left(y-\gamma^{+}>0\right) \\
&= \Delta_c E\left[\frac{\partial \gamma}{\partial r} \,\middle|\, y-\gamma>0, \gamma>0\right] Pr\left(y-\gamma>0, \gamma>0\right) \\
&= -\Delta_c E\left[\rho \mid \gamma>0, \gamma<y\right] Pr\left(\gamma>0, \gamma<y\right). \qquad (2.39)
\end{aligned}
$$

2.6.1.2 Marginal overstock costs

The expected cost of overstock at the forward production system is

$$
h_M E\left[(y-e-\zeta)^{+}\right] = h_M E\left[\left(y-\min\{\gamma^{+}, y\}-\zeta\right)^{+}\right]. \qquad (2.40)
$$

It is easy to see that the partial derivatives with respect to y and r have nonzero values only when $\gamma^{+}<y$ and $y-\gamma^{+}-\zeta>0$. Note that the former is implied by the later.

A conditional expectation of (2.40) is given by

$$
\begin{aligned}
h_M E\left[(y-e-\zeta)^+\right] &= h_M E\left[\left(y-\min\{\gamma^+, y\}-\zeta\right)^+ \mid \gamma^+ + \zeta < y\right] \\
&\quad \times Pr\left(\gamma^+ + \zeta < y\right) \\
&\quad + h_M E\left[\left(y-\min\{\gamma^+, y\}-\zeta\right)^+ \mid \gamma^+ + \zeta > y\right] \\
&\quad \times Pr\left(\gamma^+ + \zeta > y\right) \\
&= h_M E\left[y-\gamma^+ - \zeta \mid \gamma^+ + \zeta < y\right] \\
&\quad \times Pr\left(\gamma^+ + \zeta < y\right) \\
&\quad + h_M E\left[-\zeta \mid \gamma^+ + \zeta > y\right] Pr\left(\gamma^+ + \zeta > y\right)
\end{aligned} \tag{2.41}
$$

The marginal costs of overstock at the forward production system are

$$
\begin{aligned}
h_M \partial E\left[(y-e-\zeta)^+\right]/\partial y &= h_M \frac{\partial}{\partial y} E\left[y-\gamma^+ - \zeta \mid \gamma^+ + \zeta < y\right] \\
&\quad \times Pr\left(\gamma^+ + \zeta < y\right) \\
&= h_M Pr\left(\gamma^+ + \zeta < y\right)
\end{aligned} \tag{2.42}
$$

and

$$
\begin{aligned}
h_M \partial E\left[(y-e-\zeta)^+\right]/\partial r &= h_M \frac{\partial}{\partial r} E\left[y-\gamma^+ - \zeta \mid \gamma^+ + \zeta < y\right] \\
&\quad \times Pr\left(\gamma^+ + \zeta < y\right) \\
&= h_M \frac{\partial}{\partial r} E\left[y-\gamma-\zeta \mid \gamma > 0, \gamma + \zeta < y\right] \\
&\quad \times Pr\left(\gamma > 0, \gamma + \zeta < y\right) \\
&= h_M E\left[\rho \mid \gamma > 0, \gamma + \zeta < y\right] \\
&\quad \times Pr\left(\gamma > 0, \gamma + \zeta < y\right).
\end{aligned} \tag{2.43}
$$

The expected cost of overstock at the reverse production system is

$$h_R E\left[(u+e-\xi)^+\right] = h_R E\left[\left(\min\{\gamma^+, y\} - \gamma\right)^+\right]. \tag{2.44}$$

The partial derivative of (2.44) with respect to y is always zero. This follows from the observation that the coefficient of y is nonzero only when $\gamma^+ > y$ and $\gamma < y$, which happen with probability zero. To see this, let us consider the following two cases: (i) if $\gamma < 0$, then $Pr(\gamma < y < \gamma^+) = Pr(\gamma < y < 0) = 0$ and (ii) if $\gamma > 0$, $Pr(\gamma < y < \gamma^+) = Pr(\gamma < y < \gamma) = Pr(\gamma = y) = 0$. This implies that an increase in y cannot overstock the inventory of the reverse production system, which is intuitive because inventory lateral transshipment is made only to compensate the shortage for the reverse production system.

The partial derivative of (2.44) with respect to r is nonzero only when $\gamma < 0$. Let us consider the following three cases: (i) $\gamma^+ < y$ and $\gamma > 0$, (ii) $\gamma^+ < y$ and $\gamma < 0$, and (iii) $\gamma^+ > y$ and $y - \gamma > 0$. In case (i), the value of the expectation becomes zero. In case (ii), the condition is equivalent to $\gamma < 0$ because this implies $\gamma^+ = 0 < y$. This gives a nonzero coefficient of r in (2.44). The inequalities in case (iii) is equivalent to $\gamma < y < \gamma^+$ which happens with probability zero. Thus, the marginal cost of overstock with respect to r at the reverse production system is

$$\begin{aligned} h_R \frac{\partial}{\partial r} E\left[(u+e-\xi)^+\right] &= h_R \frac{\partial}{\partial r} E\left[\left(\min\{\gamma^+, y\} - \gamma\right)^+\right] \\ &= h_R \frac{\partial}{\partial r} E\left[-\gamma \mid \gamma < 0\right] Pr\left(\gamma < 0\right) \\ &= h_R E\left[\rho \mid \gamma < 0\right] Pr\left(\gamma < 0\right). \end{aligned} \tag{2.45}$$

Intuitively, overstock occurs for the reverse production system when there are more recovered products than requests for warranty service as implied by the inequality $\gamma < 0$, i.e., $\xi < u$.

2.6.1.3 Marginal shortage costs

The expected cost of shortage at the forward production system is

$$k_M E\left[(\zeta + e - y)^+\right] = k_M E\left[\left(\zeta + \min\{\gamma^+, y\} - y\right)^+\right]. \tag{2.46}$$

The marginal costs of overstock with respect to y at the forward production system are nonzero only when $\gamma^+ < y$ and $\zeta + \gamma^+ - y > 0$, i.e.,

$$\begin{aligned} k_M \frac{\partial}{\partial y} E\left[(\zeta + e - y)^+\right] &= k_M \frac{\partial}{\partial y} E\left[\left(\zeta + \min\{\gamma^+, y\} - y\right)^+\right] \\ &= k_M \frac{\partial}{\partial y} E\left[\zeta + \gamma^+ - y \mid \gamma^+ < y, \zeta + \gamma^+ - y > 0\right] \\ &\qquad \times Pr\left(\gamma^+ < y, \zeta + \gamma^+ - y > 0\right) \\ &= -k_M Pr\left(y - \zeta < \gamma^+ < y\right). \end{aligned} \tag{2.47}$$

Similarly, the marginal costs of overstock with respect to r at the forward production system are nonzero only when $\gamma^+ < y$, $\zeta + \gamma^+ - y > 0$, and $\xi > u$, i.e.,

$$k_M \frac{\partial}{\partial r} E\left[(\zeta + e - y)^+\right] = k_M \frac{\partial}{\partial r} E\left[\zeta + \gamma - y \mid \gamma > 0, y - \zeta < \gamma < y\right] \tag{2.48}$$

$$\begin{aligned} &\qquad \times Pr\left(\gamma > 0, y - \zeta < \gamma < y\right) \\ &= -k_M E\left[\rho \mid \gamma > 0, y - \zeta < \gamma < y\right] \\ &\qquad \times Pr\left(\gamma > 0, y - \zeta < \gamma < y\right) \end{aligned} \tag{2.49}$$

The negative marginal costs increase in either y or u, which helps reduce the shortage penalty incurred by the forward production system.

The expected cost of shortage at the reverse production system is

$$k_R E\left[(\xi - e - u)^+\right] = k_R E\left[\left(\gamma - \min\{\gamma^+, y\}\right)^+\right]. \tag{2.50}$$

The nonzero marginal cost of (2.50) with respect to y is obtained when $\gamma^+ > y$ and $\gamma - y > 0$, i.e.,

$$\begin{aligned} k_R \frac{\partial}{\partial y} E[(\xi - e - u)^+] &= k_R \frac{\partial}{\partial y} E[\big(\gamma - \min\{\gamma^+, y\}\big)^+] \\ &= -k_R Pr\left(\gamma^+ > y, \gamma > y\right) \\ &= -k_R Pr\left(\gamma > y\right). \end{aligned} \tag{2.51}$$

Nonzero marginal cost of (2.50) with respect to r is obtained under the same condition, i.e.,

$$\begin{aligned} k_R \frac{\partial}{\partial r} E[(\xi - e - u)^+] &= k_R \frac{\partial}{\partial r} E[\big(\gamma - \min\{\gamma^+, y\}\big)^+] \\ &= k_R \frac{\partial}{\partial r} E[\gamma - y \mid \gamma^+ > y, \gamma > y] Pr\left(\gamma^+ > y, \gamma > y\right) \\ &= -k_R E[\rho \mid \gamma > y] Pr\left(\gamma > y\right). \end{aligned} \tag{2.52}$$

2.6.1.4 Optimality conditions for (y^*, r^*)

From (2.35), (2.42), (2.47), and (2.51),

$$\begin{aligned}
\partial L(y,r)/\partial y &= \Delta_c Pr\left(\gamma > y\right) + h_M Pr\left(\gamma^+ + \zeta < y\right) \\
&\quad - k_M Pr\left(y - \zeta < \gamma^+ < y\right) - k_R Pr\left(\gamma > y\right) \\
&= \Delta_c Pr\left(\gamma > y\right) + h_M Pr\left(\gamma^+ + \zeta < y\right) \\
&\quad - k_M\left(1 - Pr\left(\gamma^+ < y - \zeta\right) - Pr\left(\gamma^+ > y\right)\right) \\
&\quad - k_R Pr\left(\gamma > y\right) \\
&= -(k_R - \Delta_c - k_M) Pr\left(\gamma > y\right) \\
&\quad + (h_M + k_M) Pr\left(\gamma^+ + \zeta < y\right) - k_M \\
&= -(k_R - \Delta_c - k_M) Pr\left(\gamma > y\right) \\
&\quad + (h_M + k_M)\left(Pr\left(\zeta < y\right) - Pr\left(y - \gamma < \zeta < y\right)\right) - k_M
\end{aligned} \tag{2.53}$$

and from (2.36), (2.43), (2.45), (2.48), and (2.52),

$$\begin{aligned}
\frac{\partial L(y,r)}{\partial r} &= -\Delta_c E\left[\rho \mid \gamma > 0, \gamma < y\right] Pr\left(\gamma > 0, \gamma < y\right) \\
&+ h_M E\left[\rho \,\middle|\, \gamma > 0, \gamma + \zeta < y\right] Pr\left(\gamma > 0, \gamma + \zeta < y\right) \\
&+ h_R E\left[\rho \,\middle|\, \gamma < 0\right] Pr\left(\gamma < 0\right) \\
&- k_M E\left[\rho \,\middle|\, \gamma > 0, y - \zeta < \gamma < y\right] Pr\left(\gamma > 0, y - \zeta < \gamma < y\right) \\
&- k_R E[\rho \mid \gamma > y] Pr\left(\gamma > y\right)
\end{aligned} \tag{2.54}$$

Using the relation

$$\begin{aligned}
E[\rho \mid \gamma > y] Pr\left(\gamma > y\right) &= E[\rho] - E[\rho \mid \gamma < 0] Pr\left(\gamma < 0\right) \\
&\quad - E[\rho \mid 0 < \gamma < y] Pr\left(0 < \gamma < y\right)
\end{aligned} \tag{2.55}$$

(2.54) can be rearranged as

$$
\begin{aligned}
\frac{\partial L(y,r)}{\partial r} &= (k_R - \Delta_c)E\left[\rho \mid 0 < \gamma < y\right] Pr\left(0 < \gamma < y\right) \\
&+ h_M E\left[\rho \;\middle|\; 0 < \gamma < y - \zeta\right] Pr\left(0 < \gamma < y - \zeta\right) \\
&- k_M E\left[\rho \;\middle|\; (y-\zeta)^+ < \gamma < y\right] Pr\left((y-\zeta)^+ < \gamma < y\right) \\
&+ (k_R + h_R)E[\rho \;\middle|\; \gamma < 0] Pr\left(\gamma < 0\right) \\
&- k_R E[\rho]
\end{aligned} \tag{2.56}
$$

The result follows from (2.53) and (2.56).

CHAPTER III

Integration of Channel Decisions in a Decentralized Closed-Loop Supply Chain With Retailer Collection Under Deterministic Non-Stationary Demands

3.1 Introduction

Recovery of postconsumer products is becoming an important strategy for realizing profitable and sustainable supply chains. The benefits of product recovery could be substantial from the perspectives of economic development, business and consumer value, and societal concern for environmental protection. Manufacturers can benefit from reduction of production costs, reduced consumption of raw-materials, enhanced corporate image, and market share protection. For example, Xerox Corporation saves hundreds of million of dollars a year by collecting, recovering, and reusing its post-consumer photocopiers (*Maslennikova and Foley*, 2000). *Kumazawa and Kobayashi* (2006) show a Life Cycle Simulation (LCS) example in which recovery of notebook computers increases business profits by 20%. Society also benefits from reduction of waste to landfills and employment growth due to the labor intensive characteristics of product recovery processes. As a result of carpet recycling efforts in the U.S., more than 1.3 billion pounds of postconsumer carpets have been diverted from landfill since 2002 (*Carpet America Recovery Effort*, 2008). Annual energy savings from product recovery are estimated at 120 trillion BTU (*Pearce*, 2009). In the UK, more than 50,000 people are employed in the remanufacturing industry, contributing about £5 billion to GDP (*Oakdene Hollins*, 2007). Examples that would best explain the benefit, as well as necessity of, product recovery are postconsumer electronic products. About 82% (1.84 million tons) of e-waste was disposed of primarily in landfills in the U.S. in 2007, with about two thirds of those in the waste stream still in working condition (*Environmental Protection Agency*, 2010; *The University of Illinois at Urbana-Champaign Sustainable Technology Center*, 2009). Whereas one ton of electronic scrap from personal computers could contain more gold than 17 tons of gold ore (*U.S. Geological Survey*, 2001), e-waste contains leachable lead, mercury, and brominated flame retardants and hence could pose a threat to water and soil through mobilized contaminants. As such, manufacturers can reclaim potential economic value as well as protect the environment by recovering postconsumer products.

Many manufacturers, however, face challenges when they engage in product recovery. In particular, they need to answer the following questions.

- How should manufacturers collect postconsumer products?
- Under what conditions manufacturers can perform profitable product recovery?
- What is the influence of market demand on decision-making associated with product recovery?

In this chapter we repeat the results reported in *Lee et al.* (2011), where we address the above problems from a two stage supply chain perspective. More specifically, we consider the optimal strategic channel decisions for a manufacturer and a retailer in a closed-loop supply chain (CLSC), i.e., supply chains for collecting and recovering postconsumer materials.

The retailer collects postconsumer products on behalf of the manufacturer throughout a product life cycle where demand first grows and then declines. Manufacturers are generally motivated to use retailers as a collection network because this avoids investment in parallel infrastructure. Therefore it is assumed the manufacturer is willing to give incentives to the retailer to participate in the reverse production activities. Similarly, the retailer can potentially benefit not only from the financial incentives offered by the manufacturer, but may also benefit from the increased foot traffic and favorable consumer perceptions.

The inclusion of return flows from retailer to manufacturer forces the consideration of a number of tradeoffs. The manufacturer may realize cost savings using return flows. However, if the retailer is the collection agent, the manufacturer cannot directly control return flows arising from the market. The only way to influence the amount of return flows is to motivate the retailer, using appropriate incentives, to collect the best level of returns for the manufacturer. Therefore, it is important for the manufacturer to influence the retailer's optimal decisions in both the reverse and forward channels in order to acquire the necessary amount of return volume. In addition, the retailer and manufacturer may pursue increased sales by utilizing return flows. The retailer is left to decide the optimal level of effort to expend on both forward and return flows, given the incentives provided by the manufacturer in pursuit of their own profit maximizing objective. A similar situation can be found in Hewlett Packard's new recycling program which uses authorized retail recycling locations for collecting end-of-life printer cartridges. Benefits include easier access to used cartridges and collection costs savings from consolidation and bulk transportation of returned cartridges. Staples, one of the HP authorized retail recycling locations, gives a store reward of $3/unit to those who return their used printer cartridge to one of its retailing network locations.

Static game vs. differential game Static models, i.e., single-period decision-making models where parameters and decision variables do not change over time, have been widely used for explaining decision-making in supply chains. However, results and insights obtained from such models do not necessarily capture important aspects of system behavior which can only be understood through dynamic models. Timely collection and recovery of postconsumer products is critical for the profitability of the entire supply chain, particularly when products exhibit short life cycle and fast depreciation, as with electronic products. In order to properly address such situations, we approach the research problems using a dynamic game model in which a manufacturer and a retailer engage in forward and reverse production activities, each pursing their own objectives. This is particularly relevant to the situations that are characterized by

- short product life cycles due to rapid advances in technology,
- massive generation of end-of-use products that contain a high level of residual value, and
- time-varying market demand that impacts interactive decision-making over time.

These are the cases where a differential game model is useful for understanding temporal aspects of optimal decision-making.

We identify profitability conditions for product recovery under time-varying market situations. Also, we will discuss how the information on the characteristics of reverse production processes can be used for finding profitable apportionment of product recovery effort among players. These new insights contribute to the enhancement of our understanding about the decision-making in closed-loop supply chains. This helps us build profitable as well as sustainable supply chains.

The remainder of the chapter is organized as follows. In section 3.2 we discuss related literature, research approach, and key assumptions. Section 3.3 reports on the solution to the model and some interpretations. In section 3.4 we draw useful insights on the decision mechanisms in closed-loop supply chains. Section 3.5 provides conclusions.

3.2 Methodology and Assumptions

Early research in product recovery in closed-loop supply chains was primarily concerned with technical issues such as disassembly, remanufacturing, and reassembly of used products. After two decades, the related fields have developed into one that encompasses diverse disciplines such as supply chain, economics, and marketing (*Guide and Van Wassenhove*, 2009). For a general review of research into sustainable product recovery, we refer readers to (*Gungor and Gupta*, 1999; *Ilgin and Gupta*, 2010).

In this chapter we attempt to obtain new insights on profitable product recovery by considering reverse production processes in two-echelon forward and reverse channels under time-varying market demands. Differential game theoretic frameworks have been used to represent conventional forward supply distribution systems, e.g., (*Pekelman*, 1974; *Eliashberg and Steinberg*, 1987; *Hartl*, 1995). *He et al.* (2007) provide an excellent review of Stackelberg differential game models in forward supply chains. In CLSC literature, *Kiesmüller et al.* (2004) address a dynamic product recovery issue involved in the case of automotive engine remanufacturing, but there exists only a single decision maker. *Savaskan et al.* (2004) analyze decentralized reverse channel structures in a static environment. However, we have not found any research that addresses the decentralized decision-making in a CLSC under time-varying environment. Our research fills this gap by extending existing studies and contributes to the literature by characterizing dynamic decision-making in CLSC under time-varying market demands. While we consider deterministic situations rather than stochastic ones to obtain analytical solutions, this would not limit the validity of interpretation of the solution on strategic level decisions.

The research is based on a decentralized two-echelon supply chain model in which the manufacturer engages in product recovery and the retailer in distribution and collection, respectively. In this chapter we consider the case where new and recovered products are assumed to be equivalent. This assumption applies to many products such as single-use cameras and printer cartridges (*Majumder and Groenevelt*, 2001). Accordingly, we describe the state of each agent using one state variable, that is, the inventory level. The dynamics of each agent's behavior is then modeled by a differential equation which represents the rate of change in inventory level over time under the retailer's re-order and manufacturer's production policy as well as exogenous variation in potential demand for the product.

Sequence of decision-making in the Stackelberg game We assume that the manufacturer leads the decision-making in the forward and reverse channels:

1. The manufacturer moves first by setting its wholesale price, collection incentive, and production quantity. The manufacturer is assumed to commit to the decision once it is made.
2. The retailer then observes the manufacturer's decision and follows by determining its retail price, buyback price, and order quantity.

The manufacturer takes into account the retailer's best reaction to its decision-making. Thus, the sequence of analysis proceeds backward as we will show in section 3.3. The model captures (i) conflict of interests between two players, (ii) external and internal time-varying dynamics, and (iii) the leader-follower relationship. A differential game (to account for (i) and (ii)) with a Stackelberg solution (for (iii)) can represent these features and is the basis for our mathematical model and analysis.

The model is depicted in Figure 3.1. There are two streams of material flows: forward flows and return flows. The forward flows are initiated by the demand signal $d(t)$ and the return flows are triggered by the incentive s_M. Forward flow originates from the supplier who ships raw materials to the manufacturer at the unit price p_S. The manufacturer receives raw materials of amount $q_M(t)$, processes the materials into finished goods incurring production cost measured by $f_M(\cdot)$, and delivers the products to the retailer at the unit wholesale price p_M. The retailer receives the

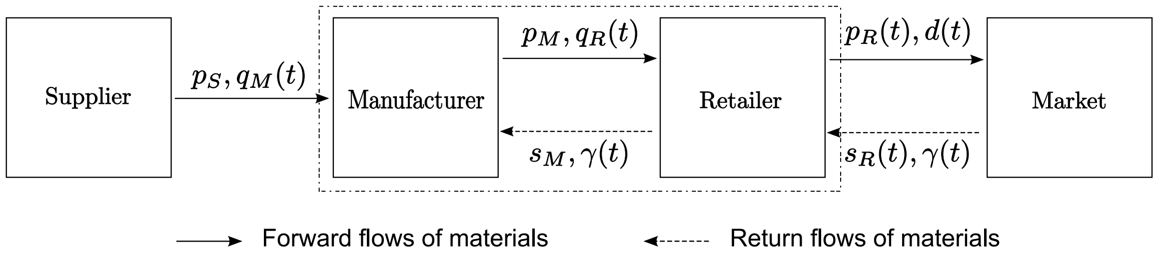


Figure 3.1: The manufacturer-retailer closed-loop supply chain model. The manufacturer moves first by determining p_M, s_M, and $q_M(t)$. The retailer then observes the OEM's decision and follows by determining $p_R(t)$, $s_R(t)$, and $q_R(t)$.

products of amount $q_R(t)$ at time t, processes them incurring handling cost $f_R(\cdot)$, and sells the final products in the market at the unit retail price $p_R(t)$. Customer demands are assumed to be immediately satisfied without backlogging. These unidirectional forward flows of materials would have been disposed of by the customer if there had not been product take-back efforts. Note that the underlying assumption for the production costs $f_R(\cdot)$ and $f_M(\cdot)$ is finite production efficiency for both firms. It is clear that production costs can be ignored when the production efficiencies are infinite.

Production take-back is initiated by the incentive signal s_M which is offered to the retailer by the manufacturer. The retailer then determines an appropriate level of take-back effort $s_R(t)$, e.g., the financial incentive given to the customer as a reward for returning a postconsumer product, to collect used products from the market. The amount of collected returns is $\gamma(t)$ and these are transferred to the manufacturer at the price of s_M for each unit of returns. Finally, the collected materials are received and reprocessed by the manufacturer and join the forward flow.

One possible issue in modeling reverse channels is the time delay which is caused by, for instance, product residence time in market. It is known that the delay in the reverse channel can be reduced by employing a product take-back program in which customers are rewarded for returning their products (*Debo et al.*, 2006). Note also that in our model the remanufactured product is a perfect substitute for the new product. Thus, we simplify the model by assuming zero delay in the reverse channel and remanufacturing.

Both firms utilize inventories to accommodate increasing demands—the so called *production smoothing* strategy. This represents a number of cases where increasing marginal costs of production prevails. *Cachon et al.* (2007) analyze unadjusted seasonal data to report that predictable demands and convex production costs are strong motivations for production smoothing. On the other hand, we shall assume stockless policies for return flows of materials. That is, return flows are assumed to be immediately transferred to the manufacturer as soon as they are collected. This reflects the lack of desirability of holding this inventory at the retailer.

The retailer's decision variables, $p_R(t)$ and $q_R(t)$ for the forward channel and $s_R(t)$ for the reverse channel, are all functions of time driven by the change in potential demands. As an example, this dynamic pricing strategy represents the situations in e-commerce retailing channels (e.g., expedia.com) where intertemporal pricing strategies have been widely adopted in recent years with advances in information technology (*Chan et al.*, 2004). On the other hand, the manufacturer's pricing decisions p_M and s_M are assumed to be constants throughout the planning horizon. This reflects the commonly practiced long-term contracts between manufacturers and retailers. Although a fixed price contract does not generally coordinate the supply chain (*Cachon*, 2003), we choose this simple contract type in order to maintain our focus on investigating the interrelated decision mechanism in forward and reverse channels. However, the manufacturer's production decision $q_M(t)$ is a function of time since the manufacturer has to respond to the time-varying re-order quantities $q_R(t)$ requested by the retailer.

The planning horizon consists of a product life cycle. We provide detailed discussion for key assumptions before presenting the formal description of the model.

Assumption III.1 (Stackelberg game)**.** *This is a non-zero-sum noncooperative dynamic game between the manufacturer and the retailer. The information structure is asymmetric and hierarchical with the manufacturer being the leader and the retailer the follower: the retailer has an a priori knowledge of the leader's decision, and the manufacturer, who has the dominant economic power over the retailer, credibly commits itself to the initial decision. Thus, both firms accept a Stackelberg equilibrium as their optimal strategies.*

The Stackelberg gaming situation is commonly observed in many supply chain systems where one player possesses dominant power over the other players, thus imposing a strategy which is favorable to itself (*Cachon and Netessine*, 2004). Depending on the situations, the manufacturer or the retailer becomes the dominant player. In this research, we choose the manufacturer as the Stackelberg leader because in typical cases, it is the manufacturer, as the final 'consumer' of the return flows, who not only decides whether or not to take-back post-consumer materials, but also performs core product recovery jobs in its facility.

In many cases, market demands exhibit an 'increasing-decreasing' pattern over time. A simple approach to modeling time-varying potential demand is to use a concave quadratic function, $a(t) = -c_1 t^2 + c_2 t + c_3$, $c_1, c_2, c_3 > 0$. The coefficients are determined by parameters of average demand and the magnitude of the overall demand change. We define the time average $\bar{a}$ and the average squared spread δ_0^2, i.e., the predictable or deterministic variation, of the deterministic non-stationary potential demands throughout the product life cycle of length T. We then solve $a'(pT) = 0$, $\bar{a} = (1/T)\int_0^T a(\tau)d\tau$, and $\delta_0^2 = (1/T)\int_0^T \left(a(\tau) - \bar{a}\right)^2 d\tau$ to determine the coefficients c_1, c_2, and c_3.

Assumption III.2 (The market)**.** *Let $\delta^2(x)$ measure the average squared spread of the time series $x(t)$ about its average $\bar{x} \equiv (1/T)\int_0^T x(\tau)d\tau$ throughout the planning horizon, i.e., $\delta^2(x) \equiv (1/T)\int_0^T \left(x(\tau) - \bar{x}\right)^2 d\tau$. The deterministic non-stationary po-*

tential demand $a(t)$ having a time average $\bar{a}$ and average squared spread δ_0^2 about $\bar{a}$ throughout the product life cycle of length T is modeled by the following quadratic concave function:[1]

$$a(t) = -\frac{3\sqrt{5}\delta_0}{T^2\omega_p}t^2 + \frac{6\sqrt{5}p\delta_0}{T\omega_p}t + \frac{\bar{a}\omega_p + \sqrt{5}(1-3p)\delta_0}{\omega_p}, \tag{3.1}$$
$$a(t) > 0, \quad 0 \leq t \leq T,$$

where $\omega_p = (15p^2 - 15p + 4)^{1/2}$ and $a(t)$ attains its maximum value at $t = pT$, $0 < p < 1$.

The market linearly responds with a constant price sensitivity factor b, $b > 0$, to the retailer's marketing efforts $p_R(t)$. The demand function $d(t)$ is characterized by

$$d(t) = a(t) - bp_R(t), \quad d(t) > 0, \quad 0 \leq t \leq T.$$

We choose a linear demand model because this makes the problem analytically tractable, especially for a Stackelberg differential game which often fails to produce a closed form solution. *Lee and Staelin* (1997) show that the specific form of the demand function, whether it is linear or nonlinear, is not very important in analyzing the hierarchical interactions within supply chains. The results obtained by a general linear model often hold for an arbitrary nonlinear model (*Milgrom*, 1994) thereby generalizing the results without involving unduly complicated analysis.

Assumption III.3 (Return flow)**.** *The amount of return flow $\gamma(t)$ is linear with a sensitivity factor β to the retailer's reverse channel marketing effort $s_R(t)$:*

$$\gamma(t) = \beta s_R(t), \quad \beta > 0. \tag{3.2}$$

[1] Potential demand $a(t)$ is the maximum demand the system can capture at time t.

We assume that the system can affect the market to generate return flows. Among those methods for encouraging consumers to bring used products back to the closed-loop supply chain, a buyback, or trade-in, program is considered to be an effective one (*Klausner and Hendrickson*, 2000) and is thus widely practiced for remanufacturing (*Guide*, 2000; *Guide et al.*, 2003). An important requirement for a return flow model is that, in a closed-loop supply chain system, the amount of returns should be less than the cumulative sales. More specifically, we can specify the condition for the current return volume such that

$$\int_{t-}^{t} \gamma(\tau)d\tau \leq \int_{0}^{t} d(\tau)d\tau - \int_{0}^{t-} \gamma(\tau)d\tau, \quad \forall t \in [0, T].$$

To implement the above idea in a manageable form, we shall assume that the potential amount of returns at time t is always less than the amount of sales at time t, for all t in $[0, T]$. This is a stricter condition because we enforce the "less-than" relationship between return flows and forward flows at each moment in time rather than in cumulative sense. Let $\rho(t)d(t)$ be the potential return flows at time t with $\rho(t)$, $0 \leq \rho(t) \leq 1$, being the correlation coefficient between the forward and return flows. The return volume is generally, especially in trade-in cases, correlated with the forward production volume. A correlated return flow model can be represented as

$$\gamma(t) = \rho(t)d(t)\frac{s_R(t)}{p_R(t)},$$

where we assume that the retailer captures the fraction, $s_R(t)/p_R(t)$, of the potential amount of return flows. Using the expression for $d(t)$, we have

$$\begin{aligned}\gamma(t) &= \rho(t)\left(a(t) - bp_R(t)\right)\frac{s_R(t)}{p_R(t)} \\ &= \rho(t)\left(\frac{a(t)}{p_R(t)} - b\right)s_R(t) \\ &\equiv \beta_o(t)s_R(t).\end{aligned}$$

The problem with the above model is that an analytical solution is intractable due to the time-varying quantity $\beta_o(t)$, which can be interpreted as the sensitivity of the return flow with respect to the buyback price at any point in time. In order to facilitate an analytical procedure, we shall replace $\beta_o(t)$ with a constant parameter $\overline{\beta}$ whose value is given by

$$\overline{\beta} = \frac{1}{T}\int_0^T \beta_o(t)dt.$$

The parameter β in (3.2) is approximated by the value of $\overline{\beta}$ which can be interpreted as the reverse flow sensitivity to the retailer's reverse channel effort $s_R(t)$. Note that if $\beta > \beta_o(t)$ for some t, then the retailer could collect more than the current demand level with $s_R(t) < p_R(t)$. Thus, we force feasibility by requiring $\beta \leq \beta_o(t)$ for all $t \in [0, T]$. In this case, the cumulative amount of returns is always less than the cumulative sales. To assume that return flows are consistent with the pattern of new sales, we choose $\rho(t) = 1$ for all $t \in [0, T]$. Then rearranging the inequality gives

$$p_R(t) \leq \frac{a(t)}{\beta + b}, \quad \forall t \in [0, T].$$

This means, as long as the retailer controls its retail price below the upper bound $a(t)/(\beta + b)$, the cumulative returns cannot exceed cumulative sales. Considering the fact that $d(t) \to 0$ as $p_R(t) \to a(t)/b$ and that $\beta \ll b$ in many real applications, an optimal retail price $p_R^*(t)$ would be found well below the upper bound $a(t)/(\beta + b)$. In its simple form, the return flow model (3.2) enables us to obtain an insightful analytical solution.

Assumption III.4 (Homogeneous product)**.** *We assume that new products and remanufactured products are homogeneous. That is, consumers do not distinguish between remanufactured and newly manufactured products, those two types of products share the same inventory space, incur the same inventory holding costs, and are sold at the same price.*

Under this assumption one variable, the inventory level, is sufficient for describing each firm's state over time. In practical situations, those two kinds of products can be homogeneous or heterogeneous depending on a variety of factors such as product category, consumer preference, and so on. Single-use cameras manufactured by Kodak would exemplify the homogeneous case. The company reports that virtually 100 percent of single-use cameras were manufactured from recycled bodies and/or parts in 2007 (*Kodak*, 2007). On the other hand, consumers can be extremely product-type conscious in their choice between new tires and retreaded tires (*Debo et al.*, 2005). A heterogeneous model should be applied in this case. *Ferguson and Toktay* (2006) address a pricing decision under the competition between new and remanufactured products. Our model takes a simpler perspective by assuming that new and recovered products are equivalent, but addresses time-dependent decision making.

Since reverse production processes are generally labor intensive, if technologies or skilled labor forces are available, it would be possible for the retailer to perform a portion of the reverse production processes, in addition to the collection of returns, in order to pursue additional revenue. This could result in production costs savings for the manufacturer because of the relieved burden of reverse production activities. The following assumption will be used for addressing such issues.

Assumption III.5 (Division of Reverse Production Processes)**.** *We assume the following:*

1. *The production capacities of both firms are measured by a common unit.*
2. *Let ϕ_R^o denote the units of production capacity the retailer uses to collect one unit of return flow when one unit of production capacity is utilized for processing one unit of forward flow. For any additional reverse production activity, the retailer uses ϕ_R, $0 < \phi_R^o \leq \phi_R$, units of production capacity to process one unit of return flow.*
3. *The manufacturer uses ϕ_M units of production capacity to process one unit of return flow when one unit of production capacity is utilized for processing of one unit of forward flow. Let ϕ_M^o, $0 < \phi_M^o < 1$, be the value of ϕ_M when the retailer's reverse production activity includes only the collection of returns.*
4. *If an additional portion of the reverse production processes is taken on by the retailer, then $\Delta\phi_M = \phi_M^o - \phi_M > 0$. This in turn determines the value of ϕ_R such that $\phi_R = g(\phi_M)$, where g is a mapping function that translates $\Delta\phi_M$ to $\Delta\phi_R$.*

When $\phi_M = \phi_M^o$ the manufacturer assumes full responsibility for the entire reverse production processes except the collection of returns which is performed by the retailer. We limit the value of ϕ_M^o to be less than one to reflect the manufacturer's wish to use less effort for the recovery of a used product than for the manufacturing of a new product.

Example III.1. Let us suppose that the production capacity of each firm is measured by 'person-hours (PH).' The manufacturer uses 10 PH to manufacture one unit of raw material into a finished good; the retailer uses 2 PH to customize a product into a final product that meets specific requirements of a customer order. The manufacturer uses 6 PH for the entire reverse production processes except the collection step, i.e.,

$\phi_M^o = 6/10 = 0.6$. In this example, the retailer's production capacity usage rate for the reverse production activities is assumed to be expressed by $\phi_R = g(\phi_M) = -6.045 + 5.075\phi_M + 1.806/\phi_M$.[2] Three examples of the division of reverse production processes are shown in Figure 3.2.

Assumption III.6 (Production costs)**.** *We assume the following:*

1. *Any available production capacity such as personnel, equipment, working time, and space can be used for processing either forward flows or return flows;*
2. *The manufacturer and the retailer have finite production efficiencies K_R and K_M (unit^2/\$), respectively, where 'unit' represents the unit of production capacity (e.g., person-hours);*
3. *The production costs of both firms are quadratic convex in the amount of production capacity utilized.*

The resulting production cost functions are characterized by

$$f_M(q_M + \phi_M\gamma) = (q_M + \phi_M\gamma)^2/K_M, \quad 0 \leq \phi_M \leq 1 \tag{3.3a}$$

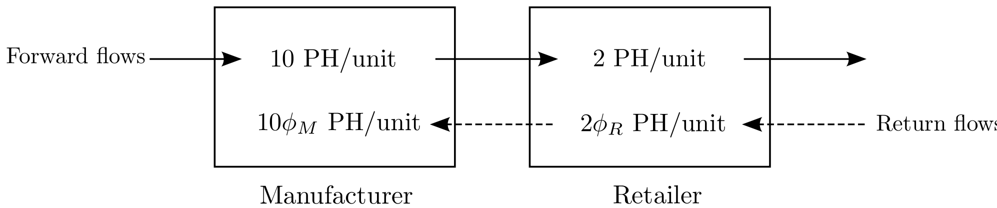
$$f_R(q_R + \phi_R\gamma) = (q_R + \phi_R\gamma)^2/K_R, \quad \phi_R \geq 0 \tag{3.3b}$$

for the manufacturer and the retailer, respectively.

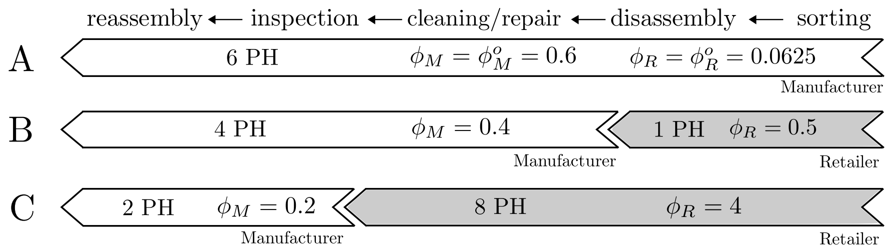
In practical situations, forward and return flows may share some, if not 100%, of production resources. Production resource sharing between forward and return flows is mentioned in, for example, *Guide* (2000) and *Bayindir et al.* (2005). According to *Debo et al.* (2006), Hewlett-Packard uses a flexible manufacturing system for new production and remanufacturing of servers.

As for the cost structure of material processing, we assume increasing marginal costs of production. Production costs often exhibit scale economies up to a certain

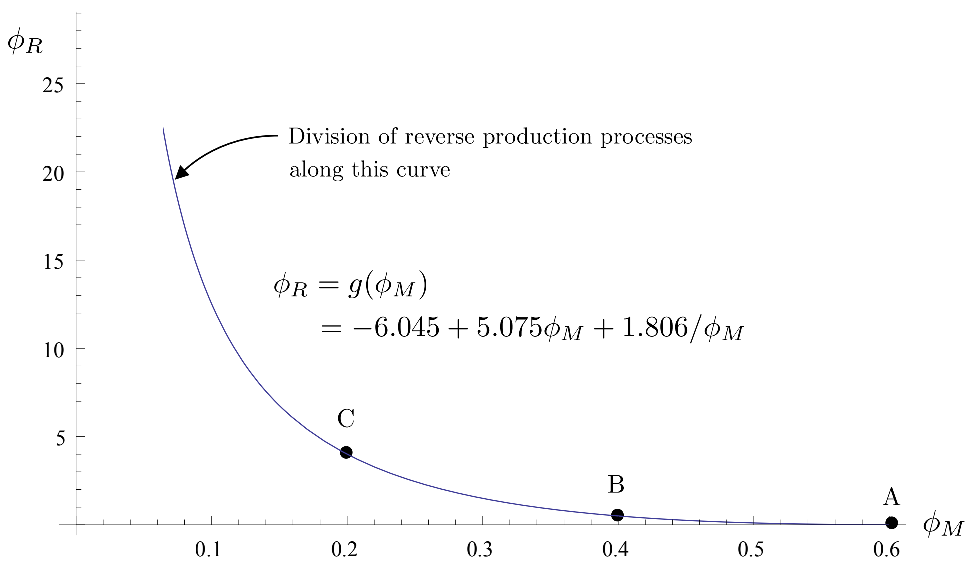
[2] The function $g(\phi_M)$ is arbitrarily chosen for an illustration.



(a) Production capacity consumption rates (PH = Person Hours)



(b) Three examples for the division of the reverse production processes



(c) Characteristic curve for dividing reverse production processes in Example III.1

Figure 3.2: Illustration of Example III.1. The earlier part (e.g., disassembly and sorting) of the above reverse production process is labor intensive and does not require advanced technology. This enables the retailer performs well for this part of the product recovery procedure. As a result, the retailer uses 1 PH for the part of reverse production process shown in case B. However, the later part (e.g., cleaning/repair) requires skilled labor and specific remanufacturing technology. Hence, in case C, the retailer needs 8 PH whereas the manufacture uses only 4 PH for the portion of the reverse production process assigned to the retailer. The curve $\phi_R = g(\phi_M)$ represents this aspect of the reverse production process.

quantity, and after this point diseconomies of scale come in effect. The key tradeoffs for both the manufacturer and retailer depend on how the production cost functions scale with throughput. In many manufacturing environments there are economies of scale with larger throughputs either because of the use of slack capacity or increased efficiencies with larger batches. We assume that these economies have already been achieved within the system and consider the case where each additional unit of production, or sales, is increasingly expensive, increasing marginal cost. This is the interesting case, because with decreasing marginal cost, the decision will be to use capacity to its maximum extent, all other things being equal. More importantly, this motivates firms to accumulate inventory to buffer future high demands. For linear production cost structures, there would be no incentive for firms to accumulate inventory because of the constant marginal costs of production.

3.3 The Model and Solution

In this section we present the optimization problems and the solutions for both firms, and make some observations about the implications of these solutions. The analysis of the Stackelberg game proceeds in the following sequence:

1. The profit maximization problem of the retailer is solved for a given wholesale price p_M and an incentive s_M. This gives a mathematical expression for the retailer's best response to the manufacturer's decision.
2. The profit maximization problem of the manufacturer is solved using the retailer's best response to obtain the optimal wholesale price p_M^*, incentive s_M^*, and production quantity $q_M^*(t)$.
3. The retailer's equilibrium solution is obtained by substituting p_M^* and s_M^* in the mathematical expression for the retailer's best response.

The Stackelberg equilibria are obtained by these three steps and are shown in section 3.3.2.

3.3.1 Retailer's problem and solution

Let the inventory level $I_R(t)$ represent the state of the retailer at time t with associated unit holding cost h_R. The retailer's profit at time t is

$$\begin{aligned} J_R(I_R, p_R, q_R, s_R, t) &= p_R d - p_M q_R - f_R(q_R + \phi_R \gamma) - h_R I_R + (s_M - s_R)\gamma \\ &= p_R\,(a - bp_R) - p_M q_R - (q_R + \phi_R \beta s_R)^2 / K_R \\ &- h_R I_R + (s_M - s_R)\beta s_R. \end{aligned}$$

The retailer wishes to maximize the total profit by controlling the retail price $p_R(t)$, re-ordering rate $q_R(t)$, and the buyback price $s_R(t)$ throughout the planning horizon under the time-varying potential demand $a(t)$. Thus, the retailer maximizes

$$\pi_R = \int_0^T J_R(I_R, p_R, q_R, s_R, t)dt \tag{3.4}$$

subject to

$$\dot{I}_R(t) = q_R(t) - d(p_R(t)) \tag{3.5a}$$

$$q_R(t) \geq 0 \tag{3.5b}$$

$$p_M \leq p_R(t) \leq \frac{a(t)}{b + \beta} \tag{3.5c}$$

$$s_R(t) \leq s_M \tag{3.5d}$$

$$I_R(t) \geq 0 \tag{3.5e}$$

$$I_R(0) = 0 \quad \text{and} \quad I_R(T) \geq 0. \tag{3.5f}$$

One of the manufacturer's decision variables, i.e., production quantity $q_M(t)$, does not appear in the retailer's model because we assume that the retailer has no concern for the production capacity of the manufacturer. In other words, it is assumed that the manufacturer can always meet the demand of the retailer, and hence the manufacturing cost is only visible to the retailer through the manufacturer's prices. The above formulation is an optimal control problem which can be solved by Pontryagin's Maximum Principle (*Pontryagin et al.*, 1962). We find the retailer's best response as follows.

Proposition III.1 (The retailer's best response)**.** *For the following nonnegative policy switching time t_R,*

$$t_R = \frac{3pT}{2} - \frac{T^2 h_R \omega_p \left(b + K'_R\right)}{4\sqrt{5}\delta_0},$$

the retailer can maximize its profit with the following two-phase best responses:

$$p^*_R(t) = \begin{cases} \frac{1}{2}\left(\frac{a(t)}{b} + h_R t\right) - \frac{1}{2}\left(\frac{a(t_R)}{b} + h_R t_R\right) + p^*_{R_2}(t_R) & 0 \le t < t_R; \\ \frac{2b + K'_R}{2(b+K'_R)}\frac{a(t)}{b} + \frac{K'_R}{2(b+K'_R)}p_M + \frac{\phi_R \beta}{2(b+K'_R)}s_M & t_R \le t \le T; \end{cases} \tag{3.6a}$$

$$q^*_R(t) = \begin{cases} \frac{K'_R h_R}{2}(t - t_R) + q^*_{R_2}(t_R) & 0 \le t < t_R; \\ \frac{K'_R}{2(b+K'_R)}\left(a(t) - bp_M - \frac{\phi_R b}{K'_R}\beta s_M\right) & t_R \le t \le T; \end{cases} \tag{3.6b}$$

$$s^*_R(t) = \begin{cases} \frac{\phi_R h_R}{2}(t_R - t) + s^*_{R_2}(t_R) & 0 \le t < t_R; \\ \frac{b + K_R}{2(b+K'_R)}s_M - \frac{\phi_R b}{2(b+K'_R)}\left(\frac{a(t)}{b} - p_M\right) & t_R \le t \le T; \end{cases} \tag{3.6c}$$

with the inventory trajectory being characterized by

$$I^*_R(t) = \begin{cases} \frac{1}{4}t^2(b+K'_R)h_R \\ +\frac{1}{2}t\left(a(t_R) - (b+K'_R)h_R t_R\right) - \frac{1}{2}\int_0^t a(\tau)d\tau & 0 \le t < t_R; \\ 0 & t_R \le t \le T; \end{cases} \tag{3.7}$$

where $K'_R \equiv K_R + \phi_R^2\beta$ *and* x_{R_i} *refers to the decision* x_R *in the ith phase policy.*

In doing so, the retailer buffers the peak demands with inventory, if the unit holding cost h_R *is bounded above such that*

$$h_R < \frac{2\sqrt{5}p\delta_0}{T\omega_p\,(b + K'_R)} \quad . \tag{3.8}$$

If (3.8) is violated, the retailer applies the second phase policy from the beginning. A larger value for δ_0, in the right hand side of (3.8) allows more expensive holding costs for the two-phase policy—a reasonable result as one of the roles of inventory is a buffer for demand variability. On the other hand, the production efficiency K_R could be high enough (i.e., low production costs) making the inventory policy less attractive compared to 'just-in-time production.' If the inventory cost is relatively expensive, the retailer may begin the operation using the second phase policy with $t_R = 0$. For those cases where the holding cost is moderate such that h_R satisfies (3.8), there exists a policy switching time t_R. We observe that throughout the two periods, the overall dynamics of $p_R^*(t)$ and $q_R^*(t)$ are primarily characterized by the market potential $a(t)$.

Competition between forward and return flows The buyback price (3.6c) follows a pattern which is opposite to that of the market potential $a(t)$. The buyback price begins linearly decreasing in the first phase and then goes in the direction opposite to the rate of change in market potential. This implies that the retailer's efforts in the forward and reverse channels are negatively correlated. The main reason for this is the finite production efficiency, which induces competition between forward and return flows within the retailer's production system. This holds for increasing or decreasing marginal costs of production, as well as for non-stationary return flow potential. One can easily identify several factors such as a low price sensitivity, a low resource price, and a large market that boosts the competitive power of forward flows. In those cases, the retailer would be more inclined to process forward flows rather than return flows simply because the former is more profitable. This tradeoff in the retailer's incentives can make it harder for the manufacturer to obtain enough returns, unless the retailer is sufficiently rewarded by the manufacturer for its efforts in the reverse channel. In other words, the retailer's view of return flow is different from the manufacturer, who has immediate incentives for obtaining a greater return volume to save manufacturing costs. This is one potential cause of difficulties for the manufacturer in increasing return volumes.

Let us now investigate the retailer's time-varying inventory policy (3.7). We first observe that the production rate should be greater than the demand rate in the beginning of the planning horizon; otherwise, some demands will be lost because initial inventory is zero for both players. Since the production rate $q_R^*(t)$ is linear in the first phase and the demand rate $d(p_R^*(t))$ follows a concave path, there must exist a time during the period of the first phase policy when $d(p_R^*(t))$ begins to exceed $q_R^*(t)$. This is necessary for the existence of policy switching time t_R. Recall that $\dot{I}_R(t) = q_R(t) - d(p_R(t))$. Therefore, the inventory path follows an increasing-decreasing trajectory up to the switching time t_R, and remains zero for the remaining time. From the solution we know that the second phase stockless policy comes in effect when cumulative quantities of production and demands become equal for the first time for $t > 0$.

One useful approach to investigate the influence of reverse production activities on the forward channel decisions is to compare the best channel decision in the CLSC to that of the counterpart system, namely the open-loop supply chain system, in which only forward production occurs. For the variable or parameter in closed-loop supply chain, say $x(\cdot)$, let $\tilde{x}(\cdot)$ denote the corresponding quantity in the open-loop system. Once we obtain $x(\cdot)$, it is straightforward to derive $\tilde{x}(\cdot)$ by setting all values of β, ϕ_R, ϕ_M, and s_M in $x^*(\cdot)$ to zero.

The policy switching time t_R can be interpreted as the length of the inventory period that begins at time 0. It is apparent from the expression of t_R that the retailer's inventory carrying period increases for a greater amount of time-varying changes, δ_0, embedded in the potential market demand $a(t)$. It is a simple matter of algebra to show that $t_R < \tilde{t}_R^0$, that is, the retailer's inventory period becomes shorter in the closed-loop supply chain. A shorter inventory carrying period for a fixed unit holding cost h_R may imply less costs, or an increase in production efficiency as we will discuss in Section 3.4.1. It can also be shown that the retailer's inventory carrying period becomes shorter for a larger return volume due to high return flow sensitivity β or a larger value of ϕ_R—the retailer's effective production efficiency K_R' increases in either case. A shorter inventory carrying period as an outcome of the reverse production activities would certainly be an incentive for the retailer to participate in the closed-loop supply chain.

3.3.2 Manufacturer's problem and solution

The manufacturer makes its decision anticipating the best response from the retailer. The manufacturer's profit at time t is characterized by

$$\begin{aligned} J_M(I_M, p_M, q_M, s_M, t) &= p_M q_R^* - p_S q_M - f_M(q_M + \phi_M \gamma(s_R^*)) \\ &\quad - h_M I_M - s_M \gamma(s_R^*) \\ &= p_M q_R^* - p_S q_M - (q_M + \phi_M \beta s_R^*)^2 / K_M \\ &\quad - h_M I_M - s_M \beta s_R^*, \end{aligned}$$

where the state variable I_M is the inventory level of the manufacturer.

Since we assume that two decision variables p_M and s_M do not change over time, we can formulate the manufacturer's problem into a two-stage optimization problem as is used by *Eliashberg and Steinberg* (1987).

$$\begin{aligned} \max_{p_M, s_M} \quad & \pi_M(p_M, s_M) \\ s.t. \quad & p_S \leq p_M \leq p_R^*, \\ & s_R^* \leq s_M \leq p_S \end{aligned} \tag{3.9}$$

where

$$\pi_M(p_M, s_M) = \max_{q_M} \int_0^T J_M(I_M, p_M, q_M, s_M, t)dt \tag{3.10}$$

subject to

$$\dot{I}_M(t) = q_M(t) + \gamma(s_R^*(t)) - q_R^*(t) \tag{3.11a}$$

$$q_M(t) \geq 0 \tag{3.11b}$$

$$I_M(t) \geq 0 \tag{3.11c}$$

$$I_M(0) = 0 \quad \text{and} \quad I_M(T) \geq 0. \tag{3.11d}$$

Solving the problem described by (3.10) and (3.11) gives the following solution.

Proposition III.2 (Optimal production quantity of the manufacturer)**.** *For the following nonnegative policy switching time* t_M*,*

$$t_M = \frac{3pT}{2} - \frac{T^2 h_M K_M \omega_p \left(b + K_R'\right)}{4\sqrt{5}\delta_0 K_R''},$$

the manufacturer can maximize its profit by adopting the following production policy:

$$q_M^*(t) = \begin{cases} \frac{1}{2}\left(K_M h_M + \phi_R \phi_M \beta h_R\right)(t - t_R) + q_{M_2}^*(t_R) & 0 \le t < t_R \\ \frac{1}{2}\left(K_M h_M\right)(t - t_M) + \frac{\phi_R \phi_M \beta \left(a(t) - a(t_M)\right)}{2(b + K_R')} + q_{M_3}^*(t_M) & t_R \le t < t_M \\ \frac{\phi_R \beta + K_R'}{2(b + K_R')} a(t) - \frac{\phi_R \beta + K_R'}{2(b + K_R')} b p_M - \frac{K_R + (1 + \phi_R) b}{2(b + K_R')} \beta s_M & t_M \le t \le T \end{cases}$$

with the optimal inventory trajectory characterized by

$$I_M^*(t) = \begin{cases} \frac{1}{4}\left(K_M h_M - K_R'' h_R\right) t^2 \\ \quad + \frac{1}{2}\left(\frac{K_R''\left(a(t_M) - a(t_R)\right)}{b + K_R'} + t_R K_R'' h_R - t_M K_M h_M\right) t & 0 \le t < t_R \\ \frac{1}{4}\left(t^2 - (t_R)^2\right) K_M h_M \\ \quad + \frac{1}{2}\left(\frac{K_R'' a(t_M)}{(b + K_R')} - t_M K_M h_M\right)(t - t_R) \\ \quad - \frac{K_R''}{2(b + K_R')} \int_{t_R}^{t} a(\tau) d\tau + I_{M_1}^*(t_R) & t_R \le t < t_M \\ 0 & t_M \le t \le T \end{cases}$$

where $K_R' \equiv K_R + \phi_R^2 \beta$, $K_R'' \equiv K_R' + (1 - \phi_M)\phi_R \beta$, and x_{M_i} refers to the decision x_M in the ith phase policy.

The manufacturer can perform production smoothing if the following condition holds

$$h_M < \frac{K_R''}{K_M} h_R. \tag{3.12}$$

Unlike the case of the retailer, the manufacturer's decision on the adoption of the above inventory policy depends upon both firms' production efficiencies. For example, if the manufacturer's production efficiency is higher than that of the retailer, then the manufacturer may be better off carrying inventories for a shorter period or none, if possible, because the manufacturer's relatively high production efficiency enables it to flexibly respond to retailer's requests incurring small production costs.

For the case where the unit holding cost h_M satisfies (3.12), the manufacturer operates the system using the above policy. In the first phase, production rate $q^*_{M_1}(t)$ is linear in the manufacturer's holding cost as well as in the retailer's holding cost. If there were no return flows, the retailer's holding cost would not affect the production rate of the manufacturer. Why do return flows bring the influence of h_R into $q^*_M(t)$ in the first phase when $0 \leq t < t_R$? This originates from two aspects of the retailer's decision mechanism. First, there are tradeoffs between inventory costs and production costs. Second, return flows compete with forward flows within the retailer's system. The retailer's holding cost in this situation plays a certain role in determining the level of its reverse channel efforts $s^*_R(t)$ as we see in (3.6c). Consequently, thus generated return flows of materials carry the influence of the inventory holding cost h_R to the manufacturer's production decision $q^*_{M_1}(t)$. Note that h_R does not appear in $q^*_{M_2}(t)$ and $q^*_{M_3}(t)$ since the retailer goes stockless for $t > t_R$.

Similar to the case of the retailer, the policy switching time t_M represents the length of the manufacturer's inventory carrying period which, as we see in the expression of t_M, increases for a greater value of δ_0. However, contrary to the case of the retailer, it can be shown that the manufacturer carries inventories for a longer period than in the case of open-loop system, i.e., $t_M > \tilde{t}^0_M$. The reason for this is that the manufacturer now faces time-varying changes coming from return flows of materials. If we let $\Delta t_M \equiv t_M - \tilde{t}^0_M$, then we can show that $\partial(\Delta t_M)/\partial\beta > 0$ and $\partial(\Delta t_M)/\partial\phi_R > 0$ for $0 \leq \phi_M \leq 1$, implying that the manufacturer's inventory period becomes longer for a larger return volume or a larger value of ϕ_R. These are the cases in which the amount of materials flowing through the manufacturer's production system exhibits greater changes over time. Note that $\Delta t_M \to 0$ and $\Delta t_R \to 0$ as $\phi_R \to 0$. In other words, there would be negligible changes in the inventory periods at both firms if the retailer's reverse production activities occupy a negligible portion of its production capacity. If inventory costs are significant, the manufacturer may find it advantageous to design a reverse channel in which the retailer performs minimal portion of reverse production processes.

In the next step, we solve the static optimization problem (3.9) for the manufacturer. We will assume an interior solution because a boundary solution (e.g., wholesale price = retail price, wholesale price = raw material price, etc.) is highly unlikely in real situations. Although boundary solutions are mathematically possible, from a practical perspective, we are more interested in an interior solution.

Proposition III.3 (Optimal contract prices)**.** *Assuming an interior solution, the optimal contract prices are*

$$p_M^* = \frac{k_1}{k} p_S + \frac{k_2}{k}\frac{\bar{a}}{b} \quad \textit{and} \quad s_M^* = \frac{k_3}{k} p_S + \frac{k_4}{k}\frac{\bar{a}}{b}$$

where

$$\begin{aligned}
k_1 &:= \beta\phi_M\left(\left(\phi_M - 1\right)\left(b + K_R\right) - b\phi_R\right) + 2K_M\left(b + K_R'\right) \\
k_2 &:= 2b\left(K_M + K_R\right) + b\beta\left(1 - \phi_M + \phi_R\right)\left(1 - \phi_M + 2\phi_R\right) \\
&\quad + \beta K_R\left(\phi_M - 1\right)^2 + 2K_M K_R' \\
k_3 &:= 2K_M\left(b + K_R'\right) + b\phi_M K_R'' \\
k_4 &:= b\left(1 - \phi_M\right)K_R'' \\
k &:= 2b\left(2K_M + K_R + \beta\left(1 - \phi_M + \phi_R\right)^2\right) \\
&\quad + 2\beta K_R\left(\phi_M - 1\right)^2 + 4K_M\left(K_R + \beta\phi_R^2\right).
\end{aligned}$$

By inspecting these expressions we find that the manufacturer does not consider the market's time-varying characteristics, δ_0^2, in deciding p_M^* and s_M^*. On the other hand, the retailer's decision is optimized for specific market conditions, i.e., both $\bar{a}$ and δ_0^2. Accordingly, the retailer is more adaptable to time-varying conditions than the manufacturer.

The retailer's equilibrium solution The retailer's equilibrium solution is obtained by substituting p_M^* and s_M^* in the retailer's best response obtained in Proposition III.1.

Proposition III.4 (The retailer's equilibrium solution)**.** *For the following nonnegative policy switching time* t_R*,*

$$t_R = \frac{3pT}{2} - \frac{T^2 h_R \omega_p \left(b + K_R'\right)}{4\sqrt{5}\delta_0},$$

the retailer can maximize its profit by adopting the following two-phase policy:

$$p_R^*(t) = \begin{cases} \frac{1}{2}\left(\frac{a(t)}{b} + h_R t\right) - \frac{1}{2}\left(\frac{a(t_R)}{b} + h_R t_R\right) + p_{R_2}^*(t_R) & 0 \le t < t_R; \\ \frac{2b + K_R'}{2(b + K_R')}\frac{a(t)}{b} + \frac{K_R'}{2(b + K_R')}\left(\frac{k_1}{k}p_S + \frac{k_2}{k}\frac{\bar{a}}{b}\right) & \\ \qquad + \frac{\phi_R \beta}{2(b + K_R')}\left(\frac{k_3}{k}p_S + \frac{k_4}{k}\frac{\bar{a}}{b}\right) & t_R \le t \le T; \end{cases} \tag{3.13a}$$

$$q_R^*(t) = \begin{cases} \frac{K_R' h_R}{2}(t - t_R) + q_{R_2}^*(t_R) & 0 \le t < t_R; \\ \frac{K_R'}{2(b + K_R')}\left(a(t) - b\left(\frac{k_1}{k}p_S + \frac{k_2}{k}\frac{\bar{a}}{b}\right)\right) & \\ \qquad - \frac{\phi_R b \beta}{2(b + K_R')}\left(\frac{k_3}{k}p_S + \frac{k_4}{k}\frac{\bar{a}}{b}\right) & t_R \le t \le T; \end{cases} \tag{3.13b}$$

$$s_R^*(t) = \begin{cases} \frac{\phi_R h_R}{2}(t_R - t) + s_{R_2}^*(t_R) & 0 \le t < t_R; \\ \frac{b + K_R}{2(b + K_R')}\left(\frac{k_3}{k}p_S + \frac{k_4}{k}\frac{\bar{a}}{b}\right) & \\ \qquad - \frac{\phi_R b}{2(b + K_R')}\left(\frac{a(t)}{b} - \left(\frac{k_1}{k}p_S + \frac{k_2}{k}\frac{\bar{a}}{b}\right)\right) & t_R \le t \le T; \end{cases} \tag{3.13c}$$

with the optimal inventory trajectory being characterized by

$$I_R^*(t) = \begin{cases} \frac{1}{4}t^2(b+K_R')h_R \\ +\frac{1}{2}t\left(a(t_R)-(b+K_R')h_R t_R\right)-\frac{1}{2}\int_0^t a(\tau)d\tau & 0 \le t < t_R; \\ 0 & t_R \le t \le T; \end{cases} \tag{3.14}$$

where $K_R' \equiv K_R + \phi_R^2\beta$, x_{R_i} *refers to the decision* x_R *in the ith phase policy, and*

$$\begin{aligned} k_1 &:= \beta\phi_M\left((\phi_M-1)(b+K_R)-b\phi_R\right)+2K_M\left(b+K_R'\right) \\ k_2 &:= 2b\left(K_M+K_R\right)+b\beta\left(1-\phi_M+\phi_R\right)\left(1-\phi_M+2\phi_R\right) \\ &\quad+\beta K_R\left(\phi_M-1\right)^2+2K_M K_R' \\ k_3 &:= 2K_M\left(b+K_R'\right)+b\phi_M K_R'' \\ k_4 &:= b\left(1-\phi_M\right)K_R'' \\ k &:= 2b\left(2K_M+K_R+\beta\left(1-\phi_M+\phi_R\right)^2\right) \\ &\quad+2\beta K_R\left(\phi_M-1\right)^2+4K_M\left(K_R+\beta\phi_R^2\right). \end{aligned}$$

In doing so, the retailer buffers the peak demands with inventory, if the unit holding cost h_R *is bounded above such that*

$$h_R < \frac{2\sqrt{5}p\delta_0}{T\omega_p\left(b+K_R'\right)} \quad . \tag{3.15}$$

Observation III.1. *For* $0 < \phi_M < 1$*, the optimal wholesale price* p_M^* *and the optimal transfer price* s_M^* *increase in the potential demand* $\bar{a}$*. However, whereas* s_M^* *increases in the resource price* p_S*, the optimal wholesale price* p_M^* *decreases under increasing* p_S *if* $k_1 < 0$*.*

It is well understood that the manufacturer would raise p_M^* under large potential demands in view of the supply-demand relation in a noncompetitive situation. It also makes sense that the manufacturer would be inclined to collect more returns by increasing s_M when the resource price rises. An interesting question is: "What would be the influence of a higher demand level on the manufacturer's reverse channel efforts?" One might think that it would be easier to collect returns as potential demand grows since the potential return volume grows as well. The manufacturer could then be able to collect the same amount of returns for less reverse channel effort s_M in a larger market. Our result is to the contrary. The manufacturer instead should increase the reverse channel effort s_M under a higher potential demand as the coefficient of $\overline{a}$ in s_M^* is positive. It can be shown from (3.6) that the retailer tends to process more forward flows when demand potential $a(t)$ increases, outweighing the ease of collection.

One might also think that the manufacturer would also raise p_M^* to recoup the rise in production costs due to an increase in resource price. However, the result demonstrates that under some conditions where $k_1 < 0$ holds, the manufacturer can lower the wholesale price even when the resource price rises. That is, firms can utilize return flows to be robust to changes in resource price. This is possible, for instance, when return flows are sufficiently responsive to the retailer's collection efforts (large value for β), the manufacturer processes more than the retailer in a reverse productions ($\phi_R \ll \phi_M$), and so on.

We have obtained optimal solutions for both firms. As an illustration, Figure 3.3 shows a numerical example for optimal decisions and inventory trajectories.

Remark III.1. The optimal solution found in this model shows zero terminal inventory, i.e., $I_M^*(T) = 0$. However, unlike the retailer, the manufacturer may find it profitable to stock non-zero terminal inventory which may be sold to other manufacturers or retailers as resource materials at the end of product life cycle. In order to address such situations we may further generalize the manufacturer's problem by allowing a nonzero terminal value for inventory. If we add a terminal value to the manufacturer's

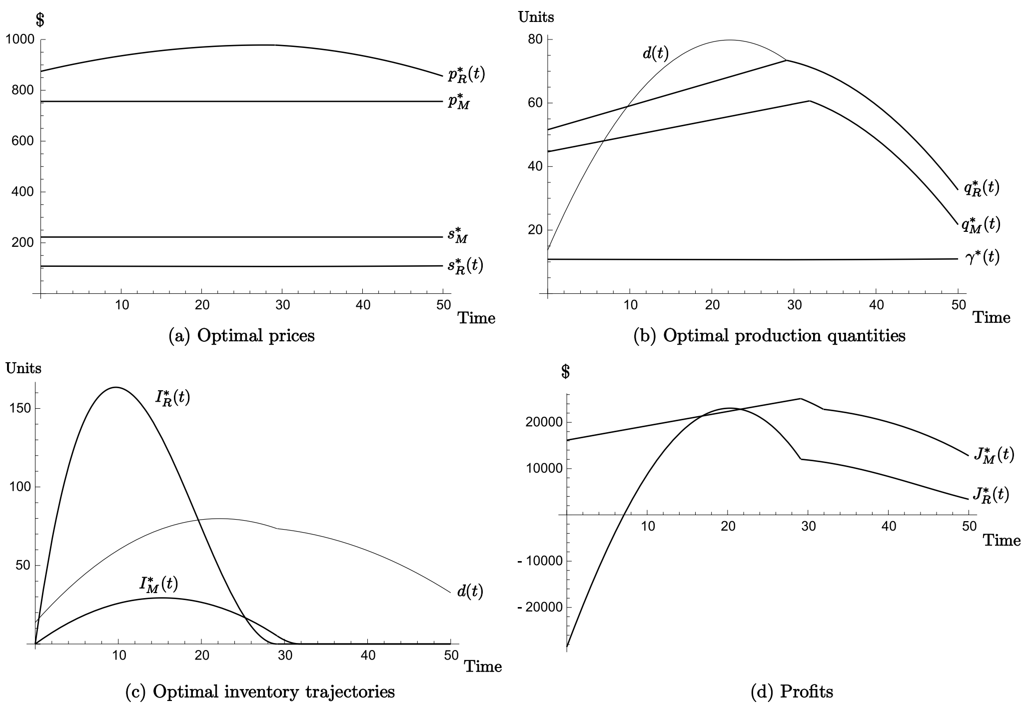


Figure 3.3: A numerical example for optimal decisions and inventory trajectories when $h_R = 1.5$, $h_M = 1$, $K_R = K_M = 1$, $\bar{a} = 1000$, $\delta_0 = 50$, $p = 0.5$, $T = 50$, $b = 1$, $p_S = 400$, $\phi_R = 0.0625$, and $\phi_M = 0.6$. Each player's decision is characterized by two-phase policy, i.e., the inventory accumulating phase and the stockless phase.

objective functional (3.10), the solution may change such that $I_M^*(T) > 0$, depending on the value of terminal inventory and the marginal cost of production. As an example, consider a terminal value $v_s I_M(T)$ for the manufacturer such that (3.10) is replaced by

$$\pi_M(p_M, s_M) = \max_{q_M} \int_0^T J_M(I_M, p_M, q_M, s_M, t)dt + v_s I_M(T),$$

where v_s is a unit salvage value. As we show in Appendix 3.6.2, we solve the dynamic optimization by using a costate variable $\lambda_M(t)$ whose value is interpreted as the shadow price of inventory. In other words, $\lambda_M^*(T)$ implies the marginal cost of accumulating additional inventory for the manufacturer at time $t = T$. This implies that if the unit salvage value v_s is larger than the cost $\lambda_M^*(T)$, then the manufacturer has an incentive to stock non-zero terminal inventory $I_M(T)$. In this case, the manufacturer employs a second policy switching time to apply a three-phase optimal policy. This may further encourage the manufacturer to collect more returns at a later stage of product life cycle, e.g., by increasing s_M^*. Since the retailer still responds to the constant contract prices p_M and s_M, this change in the manufacturer's objective would not introduce a qualitative change in the dynamic property of the retailer's optimal decision unless the salvage value v_s is made larger than the product price.

3.4 Decision Mechanisms in the Closed-Loop Supply Chain

In this section we discuss several important characteristics of optimal decisions in more detail. First, we discuss the retailer's decision mechanism of $p_R^*(t)$ and $s_R^*(t)$ to identify what changes have to be made to the retailer's decision when participating in the reverse production activities. Next, we investigate the manufacturer's decision structure to show when reverse production activity reduces the wholesale price. Finally, we highlight the characteristics of the manufacturer's resource consumption rate $q_M(t)$. We also address the characteristics of the manufacturer's production smoothing policy in the closed-loop supply chain.

3.4.1 Characteristics of the retailer's optimal solution

From (3.6), one can observe that the retailer increases both $p_R^*(t)$ and $s_R^*(t)$ when the wholesale price p_M increases.[3] The reason for this is that return flows gain competitive power as sources of additional revenue over forward flows. The retailer spends more on the reverse channel while trying to recoup the increased purchasing costs, p_M, by raising the retail price. We also observe that the retailer again increases both $p_R^*(t)$ and $s_R^*(t)$ for higher reward s_M. Higher reward would naturally motivate the retailer to exert more reverse channel effort, $s_R^*(t)$. The costs incurred by the resulting increased reverse production activities are partly recouped through the retail price.

With such interrelated outcomes for the retailer, the manufacturer's choices for p_M^* and s_M^* require more careful attention than in the case of an open-loop supply chain. We know that the manufacturer's purpose for rewarding the retailer for the collection of returns is to obtain an appropriate amount of reusable materials. However, given finite production efficiency, there would be a certain limit on the total amount of materials which can be economically handled by the retailer. Yet we may assume that the manufacturer would not want the retailer to focus excessively on reverse channel activities, if that ends up sacrificing a portion of the forward flows of materials. As such, the retailer's material processing volume would be possibly higher in a CLSC than in an open-loop supply chain. This implies that the manufacturer's decisions for p_M^* and s_M^* should be directed in such a way that the retailer's burden of material handling is well compensated.

[3] In real situations, there is some point at which the retail price does not keep increasing but stays constant when the wholesale price increases.

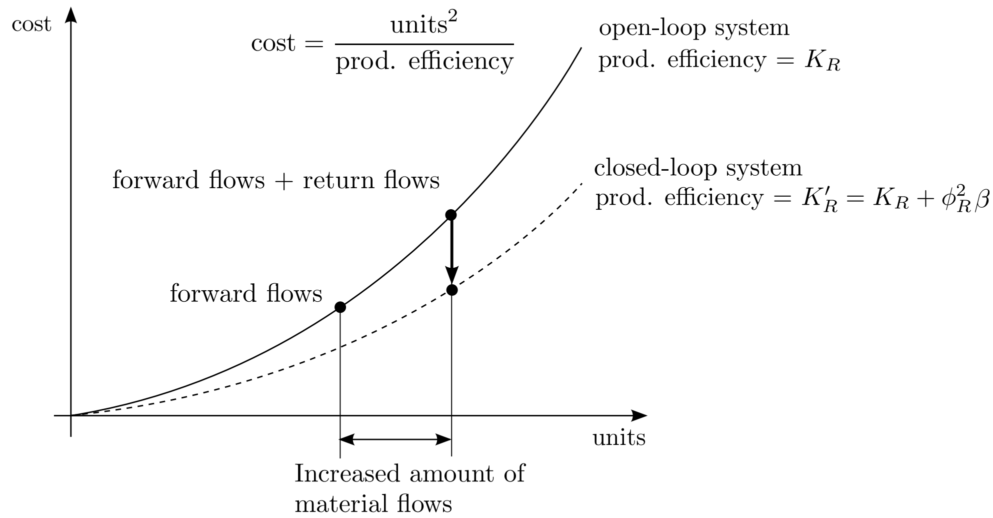


Figure 3.4: Closed-loop production efficiency K'_R for the retailer

Closed-loop production efficiency At this point, we can ask the following question: "What changes does the retailer have to make to its decisions in order to cope with the reverse production activities?" Interestingly, the retailer maintains its decisions in a similar form to that of the corresponding open-loop system as we see in the optimal solution (3.6). By comparing $p_R^*(t)$ and $\tilde{p}_R^*(t)$, we observe that the retailer's production efficiency in the CLSC appears increased from K_R to $K'_R \equiv K_R + \phi_R^2\beta$, where K_R and β have the same dimension (unit^2/\$), and ϕ_R is a dimensionless parameter. We call K'_R the *closed-loop production efficiency*. Collecting returns would result in an increased amount of materials flowing both directions through the retailer's facility which has to be more flexible to cope with such increased material flows. The term $\phi_R^2\beta$ can be regarded as the additional flexibility for the retailer's production capacity. We can then interpret K'_R as the way the retailer accounts for the additional burden of material processing in its decision making. That is, the retailer uses the closed-loop production efficiency K'_R to address the influence of reverse production activities and makes decisions in a similar way as it does in the open-loop system (Figure 3.4).

To summarize, the influence of return flows on the retail price is twofold: while the reverse production activities incur costs that increase $p_R^*(t)$, the retailer makes decisions using the closed-loop production efficiency K_R', which helps the retailer reduce $p_R^*(t)$. As such, engaging in reverse production may not guarantee a lower optimal retail price, $p_R^*(t) < \tilde{p}_R^*(t), \forall t \in [0, T]$, as one might naturally expect. Rather, two possibilities are of interest: (i) decreased $p_R(t)$ and greater demands, and (ii) increased $p_R(t)$ and fewer demands. Although net profit might be larger in either case than in the open-loop system, we are more interested in the first case for the reason that a set of optimal decisions which not only maximizes the profit of each individual player, but also generates larger demands, would be desirable from the perspective of the long-term growth and viability of the supply chain network. In light of this view, the following question naturally follows: under what conditions can firms lower their prices? To answer this question, we investigate the decision mechanism for the wholesale price which affects the retail price decision.

3.4.2 Decision mechanism of the wholesale price

We now consider the manufacturer's decision making in the CLSC. The manufacturer would wish to realize lower production costs, a lower wholesale price, and more demands, by utilizing return flows. To this end, we compare p_M^* and $\tilde{p}_M^*$ to find the conditions under which the manufacturer can achieve such objectives.

Proposition III.5. *In closed-loop operation, the manufacturer can achieve a lower optimal wholesale price,* p_M^*, $p_M^* < \tilde{p}_M^*$, *in one of the following two cases:*

$$(i) \quad 1 - 2\phi_R \frac{K_M}{K_R} - \phi_M \geq 0$$

$$(ii) \quad 1 - 2\phi_R \frac{K_M}{K_R} - \phi_M < 0, \quad \frac{(2\phi_R K_M + (\phi_M - 1)K_R)\bar{a}}{2K_M K_R + b\left(2(\phi_R + 1)K_M + \phi_M K_R\right)} < p_S$$

for $0 \leq \phi_M \leq 1$.

In order to satisfy the inequality in (i), ϕ_M and ϕ_R should be small enough for

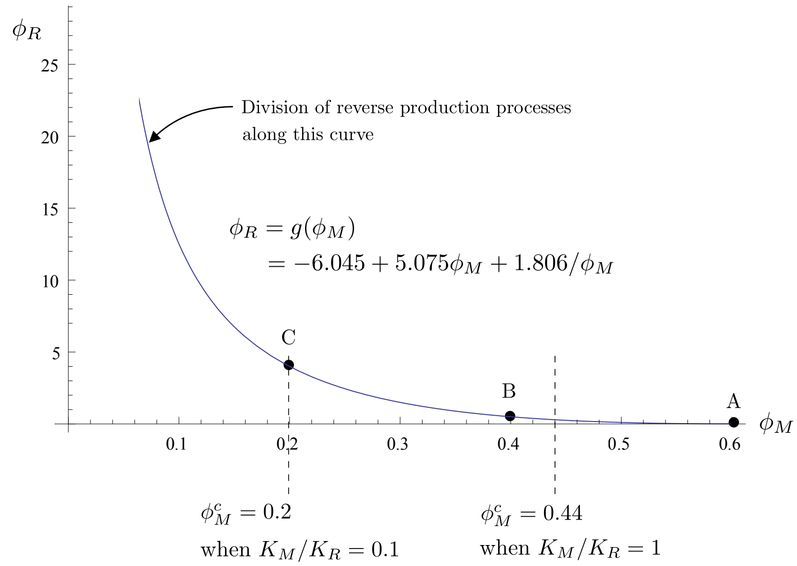


Figure 3.5: Cases where $p_M^* \leq \tilde{p}_M^*$ (continued from Example III.1). Three cases A, B, and C in Example III.1 are shown to illustrate Proposition III.5. When $K_M/K_R = 1$, the manufacturer can obtain a lower optimal wholesale price by performing at least $\phi_M^c = 0.44$ in product recovery. Among A, B, and C, case A is the only one that satisfies this condition. On the contrary, cases B and C belong to the situation where $\phi_M < \phi_M^c = 0.44$. In these cases, the manufacturer has to satisfy the inequalities in (ii) in order to obtain a lower optimal wholesale price by using return flows. However, if the retailer's production efficiency is more efficient than the manufacturer such that $K_M/K_R = 0.1$, then $\phi_M^c = 0.2$ and all three cases A, B, and C satisfy the inequality (i).

given production efficiencies K_R and K_M. If this is not the case, the manufacturer has to satisfy an additional condition as we see in (ii). As such, it depends largely on the characteristics of the reverse production processes, i.e., the mapping function $g(\cdot)$ because the value of ϕ_R is determined by $g(\phi_M)$. For ease of exposition, we shall consider three examples of division of reverse production processes in Example III.1 as are shown in Figure 3.5.

The curve $\phi_R = g(\phi_M)$ represents feasible divisions, (ϕ_M, ϕ_R), of the reverse production processes. Using this information, we can obtain a 'critical' value, say ϕ_M^c, for given values of K_R and K_M such that the inequality (i) is satisfied for $\phi_M \geq \phi_M^c$. In other words, the manufacturer's minimum level of reverse production activity is

ϕ_M^c if it wishes to obtain a lower optimal wholesale price by using return flows. If the manufacturer wants to give more reverse production activities to the retailer, i.e., $\phi_M < \phi_M^c$, then the manufacturer has to check whether the inequalities in (ii) are satisfied.

It turns out that the ratio of production efficiencies of both firms plays an important role in determining the profitable division of reverse production processes. When $K_M/K_R = 1$, $\phi_M^c = 0.44$ and only case A satisfies the inequality in (i). This is due to the characteristics of the hypothetical reverse production processes which require more skilled labor or high technology in later stage of the processes. The retailer has to use more production resources in cases B and C. Under such a situation, the inequalities in (ii) have to be satisfied for the return flow to become competitive enough over the forward flow so that there is a sufficient amount of return volume that enables the manufacturer to obtain a lower optimal wholesale price. On the other hand, the retailer will be able to economically accommodate more reverse production processes if its production efficiency is high. As the retailer becomes more efficient, i.e., a decrease in the ratio K_M/K_R, the value of ϕ_M^c becomes smaller and more cases satisfy the inequality in (i).

The example shown in Figure 3.5 belongs to an intermediate case of two extremes shown in Figure 3.6. In one extreme case $g(\phi_M)$ approaches an 'L' shape such that the retailer is efficient for the majority of the reverse production process. This would be the cases where product recovery is simple. For example, more and more major cell phone carriers (e.g., AT&T, Verizon, etc.) refurbish and resell cell phones. In this case, retailers are more efficient in product recovery activities such as program reset and repackaging than manufacturers. Reverse material flows may not need to proceed up to the manufacturer because most of the product recovery processes can be economically performed by the retailer. In the other extreme $g(\phi_M)$ approaches a '⅂' shape such that the manufacturer is efficient for the majority of the reverse production

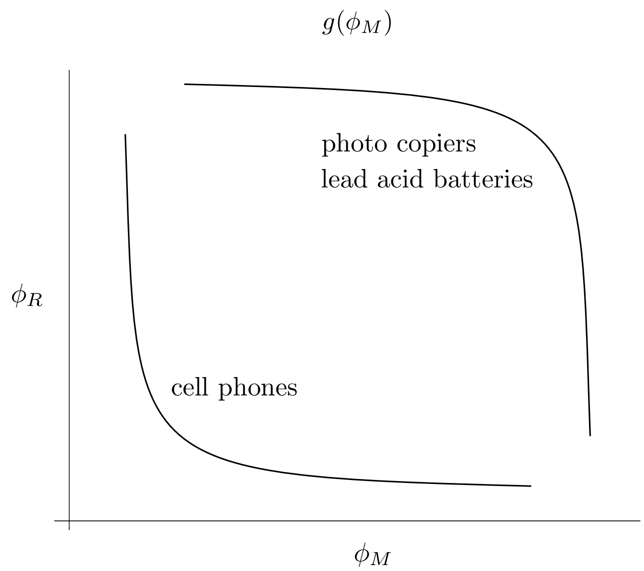


Figure 3.6: Two extreme cases of $g(\phi_M)$. More and more major cell phone carriers (e.g., AT&T, Verizon, etc.) refurbish and resell cell phones. In this cases, retailers can efficiently handle the entire product recovery processes. On the contrary, recovery of photo copiers and lead acid batteries require specific technique and equipments which may only be economically operated by manufacturers. For intermediate cases, the model can be used to find a profitable division of reverse production process for a given $g(\phi_M)$.

process. Product recovery which requires specialized technology, equipment, or labor would belong to this case. An example is the recycling of lead-acid batteries. The recycling rate of lead-acid batteries is more than 98% in the U.S., but the recycling process may not be safely and economically performed by the retailer due to sulfuric acid and lead, both of which are toxic. Another example is the recovery of used photo copiers where manufacturers are responsible for most of the recovery processes such as disassembly, cleaning, inspection, sorting, reconditioning, and reassembly. Whereas the division of recovery processes would be obvious for such extreme cases, the approach in this research can be useful for finding a profitable apportionment of product recovery effort among participating firms, especially for intermediate cases where an optimal allocation of reverse production activities is not obvious.

3.4.3 Resource input stream $q_M^*(t)$

One of the purposes of using return flows is to reduce raw material consumption, which is represented by the quantity $q_M^*(t)$ in our model. While we pursue a lower level of raw material consumption in the closed-loop supply chains, we are also interested in the qualitative change in $q_M^*(t)$, i.e., the relative change in the time-varying dynamic property, since this could influence the production plans of the manufacturer as well as the supplier. In order to investigate the characteristics of the time-varying dynamics of the resource input stream $q_M^*(t)$, we shall use the measure $\delta^2(q_M)$. Recall that the value $\delta^2(x)$ represents the extent of changes that $x(t)$ undergoes over time. For instance, we constructed the market potential $a(t)$ in (3.1) such that $\delta^2(a) = \delta_0^2$.

Let us first consider the period $[0, t_R]$. In this period, q_M^* and q_R^* are straight lines and $\delta^2(q_M^*)$ and $\delta^2(q_R^*)$ are determined by the slopes $K_M h_M + \phi_R \phi_M \beta h_R$ and $K_R h_R + \phi_R^2 \beta h_R$, respectively. For instance, we have $\delta^2(q_M^*) < \delta^2(q_R^*)$ for $K_M \approx K_R$, $h_M < h_R$, and $\phi_R \ll \phi_M$. Note that we also have $\delta^2(\tilde{q}_M^*) < \delta^2(\tilde{q}_R^*)$ in this case.

Contrary to this, in the time interval $[t_M, T]$, we find that $q_M^*(t)$ undergoes change such that for $\phi_R > 0$,

$$\frac{\delta^2(q_M^*)}{\delta^2(q_R^*)} = \left(\frac{K_R + \phi_R^2 \beta + \phi_R \beta}{K_R + \phi_R^2 \beta} \right)^2 > 1.$$

Differentiation with respect to each parameter reveals that the above ratio (i) increases in return flows sensitivity β, (ii) decreases in the retailer's production efficiency K_R, and (iii) increases in the retailer's burden ϕ_R when $\phi_R < \sqrt{K_R/\beta}$. If there are no return flows (i.e., $\phi_R = 0, \beta = 0$), we have $\delta^2(\tilde{q}_M^*) = \delta^2(\tilde{q}_R^*)$. So we have $\delta^2(q_M^*) > \delta^2(q_R^*)$ only when return flows exist in the system, and the ratio $\delta^2(q_M^*)/\delta^2(q_R^*)$ depends on how the manufacturer contracts with the retailer with respect to the reverse production activities, i.e., the dividing point (ϕ_M, ϕ_R) of the reverse production process on the characteristic curve $g(\phi_M)$. For instance, if the manufacturer burdens the retailer more (i.e., higher value for ϕ_R) but no more than the threshold value $\sqrt{K_R/\beta}$, then return flows become less attractive for the retailer giving more competitive power to forward flows within the retailer's production system. This not only shrinks the return volume, but also increases the value of $\delta^2(q_M^*)/\delta^2(q_R^*)$—both are certainly undesirable results for the system.

This aspect can be explained by the fact that the retailer makes its decisions in such a way that forward and return flows of materials are negatively correlated. Recall that the manufacturer's goal is to replace a portion of the resource input streams by return flows in order to reduce production costs. This is equivalent to subtracting the return volume, $\gamma^*(t)$, from the retailer's order amount, $q_R^*(t)$, and by doing so the two flows become positively correlated while they are being merged to form the resource input stream $q_M^*(t)$. As a result, $q_M^*(t)$ shows a greater time-varying dynamics than $q_R^*(t)$ while exhibiting a smaller time average amount. This is not a favorable situation for the supplier because it has to operate with a reduced, but more time-varying, order amount. It is thus possible that such changes in $q_M^*(t)$ persist over multiple product cycles so that the system experiences difficulty in establishing stable flows of raw materials. This implies the need for a supplier capable of flexible production or asynchronous material flows control that eliminate the amplification of time-varying change in $q_M^*(t)$.

3.5 Conclusions

This research extends the forward supply chain dynamic model of *Eliashberg and Steinberg* (1987) by considering the return flows of materials. The solutions for the forward channel decisions are analogous, except that the manufacturer orders less from the supplier because there is a return flow from the market. Our model is distinguished by the existence of reverse channel decisions that are integrated with forward channel decisions. This provides new insights for decision-making in a closed-loop supply chain where two decision makers, a manufacturer and a retailer, interact over time. For example, the differential game model tells us the optimal level of activity for the retailer in both forward and reverse channels as the market demands change over time. Such adaptive behavior of the retailer affects the profitability and the collection rate (i.e., sustainability) of the entire supply chain. Product recovery has higher benefit for products having short life cycles with substantial level of residual values at the end of life. The model developed in this chapter can be used to understand the characteristics of optimal decisions in closed-loop supply chains subject to rapid changes in market demands.

We find that the retailer's optimal decision is characterized by generating return flows which are negatively correlated with the forward flows. This property, however, induces competition between forward and return flows within the retailer's facility thereby making the players in the system seek optimal interrelated solutions by balancing tradeoffs.

The results indicate that the manufacturer and the retailer can pursue profitable reverse production by appropriately sharing the processing tasks by considering the characteristic curve $g(\phi_M)$. One of the main advantages is an expansion in market demands due to a lower wholesale price and a greater return volume. This complements the advantages of utilizing the retailer's already established infrastructure in forward channels as a reverse production channel.

We have shown that a CLSC can accommodate extensive product recovery processes when it is easy to obtain postconsumer materials. This shows the potential opportunity for profitable closed-loop supply chains in some categories of products even when they require relatively high costs of product recovery.

The approach employed in this research has some limitations even though it provides insight. For strategic level decisions, the current model does not consider uncertainty. Uncertainty will influence the dynamics of return flows and hence the decision making in closed-loop supply chains. The added complexity of modeling uncertainty makes it difficult to obtain results in the differential game framework, and it is not clear that specific insights can be established for these cases. A simulation-based approach may be appropriate for these systems, once reasonable parameter ranges have been established through deterministic modeling.

Further research We consider several modifications to the present system as ways to resolve the issue of the competition of materials flows. First of all, a third-party-collecting system or a manufacturer-collecting system could resolve the issue of the interference of forward and return flows at the front end of the supply chain system. However, *Savaskan et al.* (2004) report that in a decentralized supply chain, in which all participants pursue their own profit and have sufficient production capacity, the retailer is most preferred in performing product take-back from the market. Our approach is mainly distinguished from *Savaskan et al.* (2004) by considering convex production costs in a dynamic setting. Hence, comparing the performance of those different reverse channel structures within our framework would be an interesting extension. It would be also interesting to see what happens if we include factors other than net profit, e.g., return volume, in the objective function.

Alternatively, we can separate the retailer's joint production system into two independent production systems—one for forward production, and the other for reverse production. More specifically, we can use $f_R(q_R)/K_R + f_R(\phi_R\gamma)/K_R$ instead of $(q_R + \phi_R\gamma)^2/K_R$ as a new production cost model for the retailer. This decouples the cost of handling forward and return flows, and hence eliminates the competition within the retailer's production system. As a result the cost always decreases, e.g., $(q_R + \phi_R\gamma)^2/K_R > q_R^2/K_R + (\phi_R\gamma)^2/K_R$, which makes the results substantially different from the model presented in this chapter and out of its scope.

3.6 Appendix

3.6.1 Proof of Proposition III.1

Introducing an adjoint variable $\lambda_R(\cdot)$, the retailer's Hamiltonian is expressed as

$$\begin{aligned} H_R(I_R, p_R, q_R, s_R, \lambda_R, t) &= J_R(I_R, p_R, q_R, s_R, t) + \lambda_R(q_R - d(p_R)) \\ &= p_R d(p_R) - p_M q_R - f_R(q_R + \phi_R \gamma) \\ &- h_R I_R + (s_M - s_R)\gamma(s_R) \\ &+ \lambda_R(q_R - d(p_R)). \end{aligned} \tag{3.16}$$

The state variable is constrained by a non-negativity condition and the control variables are bounded (3.5). For simplicity, we shall consider interior solutions. We then formulate the Lagrangian as follows:

$$L_R = H_R + \rho_R I_R \tag{3.17}$$

where ρ_R is a Lagrangian multiplier.

Our solution approach is to segment the trajectory of I_R into constrained ($I_R = 0$) and unconstrained ($I_R > 0$) pieces, and join them at each junction point using jump conditions (*Knowles*, 1981). Since the firms face increasing demand in the beginning, we start with an unconstrained subproblem, i.e., $I_R > 0$.

Subproblem 1 ($I_R > 0$). Let us assume $I_R(t) > 0$ for $t \in [0, t_R)$. By complementary slackness, we have $\rho_R(t) = 0$. The first order necessary conditions are

$$\partial L_R/\partial p_R = a(t) - 2bp_R(t) + b\lambda_R(t) = 0 \tag{3.18a}$$

$$\partial L_R/\partial q_R = -f_R'\left(q_R(t) + \beta\phi_R s_R(t)\right) - p_M + \lambda_R(t) = 0 \tag{3.18b}$$

$$\partial L_R/\partial s_R = -\beta\phi_R f_R'\left(q_R(t) + \beta\phi_R s_R(t)\right) + \beta\left(s_M - s_R(t)\right) - \beta s_R(t) = 0. \tag{3.18c}$$

The adjoint equation is

$$\dot{\lambda}_R(t) = -\partial L_R / \partial I_R = h_R. \tag{3.19}$$

Substituting (3.1), (3.2), and (3.3) into (3.18), we obtain the following stationary points

$$p_R^* = \frac{1}{2}\left(\frac{a(t)}{b} + \lambda_R(t)\right) \tag{3.20a}$$

$$q_R^* = \frac{1}{2}\left(K_R + \beta\phi_R^2\right)\left(\lambda_R(t) - p_M\right) - \frac{1}{2}\beta s_M \phi_R \tag{3.20b}$$

$$s_R^* = \frac{p_M \phi_R}{2} + \frac{s_M}{2} - \frac{1}{2}\phi_R \lambda_R(t). \tag{3.20c}$$

Recall that $f_R(q_R)$ is a strictly increasing convex function. Hence, the interior solution (3.20b) is unique. We also find that $f_R(\cdot)$ and return flows sensitivity β does not appear in p_R^* and s_R^*, and that the adjoint variable $\lambda_R(t)$ influences the dynamic properties of p_R^* and s_R^* with opposite signs. This leads to the following observation: the retailer's reverse channel effort $s_R^*(t)$ is negatively correlated with its forward channel effort $p_R^*(t)$ as long as it processes forward and return flows of materials within the same production system.

Note that the stationary point (3.20) is the only one of its kind that satisfies (3.18) and is the maximum point in the interior of the control space because of the convexity of the Hamiltonian (3.16): $\partial^2 H_R / \partial q_R^2 = -2/K_R$, $\partial^2 H_R / \partial p_R^2 = -2b$, and $\partial^2 H_R / \partial s_R^2 = -2\beta^2 / K_R - 2\beta$, all of which, with $K_R > 0$, $b > 0$, and $\beta > 0$, are negative.

At this point, it remains to characterize the adjoint variable λ_R in order to obtain the optimal controls q_R^* and p_R^*. For this non-binding segment, (3.19) has the solution

$$\lambda_R(t) = h_R t + C_R \quad t \geq 0 \tag{3.21}$$

where C_R is a constant. In order to determine the value of C_R, we solve the next subproblem where the state constraint is active, and then apply the jump condition on the adjoint variable $\lambda_R(t)$ at $t = t_R$.

Subproblem 2 ($I_R(\cdot) = 0$). For this subproblem, we allow nonzero value for $\rho_R(t)$. In accordance with the restricted maximum principle (*Knowles*, 1981), the main concern here is to control the system such that $I_R(t) = 0$. The necessary and sufficient condition is to make the normal vector on the surface of state constraint perpendicular to the velocity of state trajectory, i.e.,

$$q_R(t) - d(p_R(t)) = 0. \tag{3.22}$$

With the additional constraint (3.22) to the maximization of the Lagrangian (3.17), it is straightforward to see that the optimal control for this constrained trajectory has the same form (3.20) as is obtained in the previous subproblem. The adjoint variable λ_R, however, is different from (3.21). Solving (3.22) for $\lambda_R(t)$, we obtain

$$\lambda_R(t) = \frac{a(t) + \beta\phi_R s_M + (K_R + \beta\phi_R^2)\, p_M}{b + K_R + \beta\phi_R^2} \tag{3.23}$$

for $t \geq t_R$.

We determine the values of C_R using the jump condition $\lambda_R(t_R-) = \lambda_R(t_R+)$ at the entry point on the state boundary. Equating (3.21) and (3.23) at time t_R, and solving for C_R, we obtain

$$C_R = \lambda_R(t_R+) - t_R h_R. \tag{3.24}$$

Further, we determine the value of switching point t_R using the fact that $I_R(0) = 0$

and $I_R^*(t_R) = 0$:

$$\begin{aligned} 0 &= I_R^*(t_R) - I_R^*(0) \\ &= \int_0^{t_R} \dot{I}_R^*(\tau) d\tau \\ &= \int_0^{t_R} (q_R^*(\tau) - d(p_R^*)) d\tau. \end{aligned} \tag{3.25}$$

Solving (3.25) for t_R gives

$$t_R = \frac{3pT}{2} - \frac{T^2 h_R \omega_p (b + K_R')}{4\sqrt{5}\delta_0}.$$

We assume that the retailer wishes to perform production smoothing and has no concern for inventory storage capacity during high demands times. It can be shown that demands hit the peak at $t = t'$ for which $\dot{a}(t') = 0$. Solving for t' and rearranging the inequality $t_R > t'$ we obtain (3.8).

Finally, with $I_R(t) = 0, t \in [t_R, T]$, the transversality condition $\lambda_R(T) I_R(T) = 0$ is satisfied at the end of this constrained segment. Therefore, two subproblems characterize the optimal control and trajectory of the retailer's problem.

3.6.2 Proof of Proposition III.2

We first solve the optimal control problem (3.10), and then the static optimization problem (3.9). The structure of the manufacturer's state constrained optimal control problem is analogous to that of the retailer. For the similar reason, we will break the optimal control problem into subproblems starting with an unconstrained trajectory.

Subproblem 1 ($I_M > 0$). Let us assume $I_M(t) > 0$ for $t \in [0, t_M)$. Using an adjoint variable $\lambda_M(\cdot)$, the manufacturer's Hamiltonian is given by

$$\begin{aligned} H_M(I_M, q_M, \lambda_M, t) &= J_M(I_M, p_M, q_M, s_M, t) + \lambda_M(q_M + \gamma^* - q_R^*) \\ &= p_M q_R^* - p_S q_M - f_M(q_M + \phi_M \gamma^*) - h_M I_M \\ &- s_M \gamma^* + \lambda_M(q_M + \gamma^* - q_R^*). \end{aligned} \tag{3.26}$$

Assuming interior solutions, we formulate the Lagrangian as follows:

$$L_M = H_M + \rho_M I_M \tag{3.27}$$

where ρ_M is a Lagrangian multiplier whose value is zero in this non-binding segment.

The first order condition is

$$\frac{\partial L_M}{\partial q_M} = -p_S - f_M'(q_M) + \lambda_M = 0 \tag{3.28}$$

the adjoint equation is

$$\dot{\lambda}_M = -\frac{\partial L_M}{\partial I_M} = h_M. \tag{3.29}$$

Solving (3.28) for q_M gives

$$q_M^* = \frac{1}{2}\left(K_M(\lambda_M(t) - p_S) - 2\beta\phi_M s_R(t)\right). \tag{3.30}$$

The second partial derivatives $\partial^2 L_M / \partial q_M^2 = -2/K_M$ is negative with $K_M > 0$, so the Lagrangian is concave over the control space. Hence, (3.30) maximizes the Lagrangian (3.27).

It remains to determine the adjoint variable $\lambda_M(\cdot)$. Integrating (3.29) gives

$$\lambda_M(t) = h_M t + C_M \quad t \geq 0, \tag{3.31}$$

where C_M is a constant to be determined using the jump condition with the next subproblem.

Subproblem 2 ($I_M = 0$). In this constrained segment, the following equality constraint makes the state variable remain at the state boundary:

$$q_M(t) + \gamma(s_R^*(t)) - q_R^*(t) = 0. \tag{3.32}$$

We will maximize the Lagrangian (3.27) as before but with the above additional constraint (3.32). Using (3.20b), and $q_M^* = q_R^* - \gamma(s_R^*)$ from (3.32), and solving for $\lambda_M(t)$ we obtain

$$\begin{aligned}
\lambda_M(t) = & \frac{a(t)\left(K_R + \beta\phi_R\left(1 - \phi_M + \phi_R\right)\right)}{K_M\left(b + K_R + \beta\phi_R^2\right)} \\
& - \frac{bp_M\left(K_R + \beta\phi_R\left(1 - \phi_M + \phi_R\right)\right)}{K_M\left(b + K_R + \beta\phi_R^2\right)} \\
& - \frac{\beta s_M\left(K_R + b\left(1 - \phi_M + \phi_R\right) - K_R\phi_M\right)}{K_M\left(b + K_R + \beta\phi_R^2\right)} + p_S, \qquad t \geq t_M.
\end{aligned} \tag{3.33}$$

By the jump condition, the adjoint variable $\lambda_M(t)$ is continuous at the entry point t_M. Equating (3.31) and (3.33) at $t = t_M$ gives

$$C_M = \lambda_M(t_M+) - t_M h_M.$$

In order to determine the switching time t_M, we solve the following integral equation

$$
\begin{aligned}
0 &= I_M(t_M) - I_M(0) \\
&= \int_0^{t_M} \dot{I}^*_M(\tau) d\tau \\
&= \int_0^{t_R} \left(q^*_{M_1}(\tau) - q^*_{R_1}(\tau) + \gamma(s^*_{R_1}(\tau))\right) d\tau \\
&\quad + \int_{t_R}^{t_M} \left(q^*_{M_1}(\tau) - q^*_{R_2}(\tau) + \gamma(s^*_{R_2}(\tau))\right) d\tau
\end{aligned}
\tag{3.34}
$$

Solving (3.34) for t_M gives

$$
t_M = \frac{3pT}{2} - \frac{T^2 h_M K_M \omega_p \left(b + K'_R\right)}{4\sqrt{5}\delta_0 \left(K'_R - \beta\phi_M\phi_R + \beta\phi_R\right)}.
$$

Since the retailer accumulates and discharges inventory during its inventory period $[0, t_R]$, the manufacturer would follow a similar pattern of inventory policy during its inventory period $[0, t_M]$. One way to make this possible is to have the manufacturer's inventory peak time in the time interval $[0, t_R]$. Note that $q^*_{M_1}(\tau) - q^*_{R_1}(\tau) + \gamma(s^*_{R_1}(\tau))$ is a straight line and thus we require its slope to be negative in $[0, t_R]$ such that the rate of inventory accumulation changes from positive to negative during this period. After some algebraic steps we obtain (3.12). Alternatively, we can use the inequality $t_R < t_M$ to obtain the same result.

The transversality condition is $\lambda_M(T)I_M(T) = 0$ which is satisfied by the current solution. This completes the characterization of the optimal control q^*_M and the optimal inventory trajectory I^*_M of the manufacturer.

3.6.3 Proof of Proposition III.3

Static Optimization Assuming an interior solution, we perform the second derivative test for the existence and uniqueness of a relative maximum. It is an algebraic procedure to show that $(\partial^2\pi_M/\partial p_M^2)(\partial^2\pi_M/\partial s_M^2) - \partial^2\pi_M/(\partial p_M \partial s_M) > 0$ and

$\partial^2\pi_M/\partial p_M^2 < 0$. Thus, we have a relative maximum. Equating $\partial\pi_M/\partial p_M$ and $\partial\pi_M/\partial s_M$ to zero gives the solution.

3.6.4 Other Proofs

3.6.4.1 Proof of Observation III.1

Using the notation of Proposition III.3, it can be shown that $\partial p_M^*/\partial\bar{a} = k_2/bk > 0$, $\partial s_M^*/\partial\bar{a} = k_4/bk > 0$, and $\partial s_M^*/\partial p_S = k_3/k > 0$ for $0 \leq \phi_M \leq 1$. We have $\partial p_M^*/\partial p_S = k_1/k < 0$ if

$$\frac{2K_R\left(b + K_R'\right)}{\left(1-\phi_M\right)\left(b+K_R\right) + b\phi_R} < \beta\phi_M$$

for $0 \leq \phi_M \leq 1$.

3.6.4.2 Proof of Proposition III.5

We first obtain $\tilde{p}_M^*$ by setting the values of β, ϕ_R, and ϕ_M to zero, and then the difference $p_M^* - \tilde{p}_M^*$ is computed. It is a matter of algebra to show that the sign of $p_M^* - \tilde{p}_M^*$ is determined by the following

$$-\bar{a}\left(K_R - K_R\phi_M - 2K_M\phi_R\right) - p_S\left(2bK_M\phi_R + bK_R\phi_M + 2bK_M + 2K_MK_R\right).$$

Depending on the sign of $K_R - K_R\phi_M - 2K_M\phi_R$ we have two cases. The result follows.

CHAPTER IV

Risk Aversion and Product Cannibalization in Closed-Loop Supply Chains

4.1 Introduction

In closed-loop supply chains (CLSCs), manufacturers take back postconsumer products, recover the residual values through remanufacturing, and resell the products and new products in a market. Since the remanufactured product is generally sold at a lower price than the new product, there is a widespread belief that the demand for the remanufactured product is gained at the cost of new product sales. This is so-called *product cannibalization* that concerns manufacturers. Manufacturers try to protect new product sales from the cannibalizing effect of remanufactured products. For instance, Alpha Equipment destroys most trade-in returns, the value of which exceeds $800 million in total, to prevent potential market cannibalization (*Atasu et al.*, 2010). Bosch Power Tools introduces remanufactured products only where the company has less than 50% market share to minimize the impact on the sales of new products (*Ferguson*, 2009). The effort to avoid product cannibalization is largely based on common sense rather than a scientific basis. CLSC research is still in its infancy with regard to this important problem. The lack of understanding of product cannibalization has made manufactures reluctant to introduce remanufactured products into the market.

In this chapter, we challenge the common perception about product cannibalization by asking the following questions: Is product cannibalization necessarily bad?

Is the degree of product cannibalization an appropriate measure for evaluating a company's performance? In what situation is product cannibalization more likely? In order to properly address these questions, we develop a model considering the following aspects of CLSCs.

- Consumers generally have low valuations for remanufactured products.
- The sales volume of remanufactured products is limited by the amount of collected postconsumer products.
- The collection quantity depends on the buyback price if postconsumer products are collected through a take-back program.
- There exists a significant level of uncertainty in the quality of postconsumer products.

The major contribution of this research lies in providing new insights on interrelationship among market segmentation characteristics, quality uncertainty in remanufactured products, consumers' risk averse decision-making, firm's risk averse pricing decisions, product cannibalization, and the total profit of the system. Based on quantitative models, we show that a manufacturer's profit generally increases with remanufacturing, but this accompanies product cannibalization. Increasing uncertainty in the quality of a remanufactured product reduces consumers' willingness to buy the product. This helps reduce the degree of product cannibalization, but the company will experience decreased profit. The best case is found when consumers are highly likely to return their used products. We confirm that the uniform distribution assumption gives qualitatively similar results to more general distributions, but there can be substantial differences in the profit that can be generated. In general, a more refined understanding of the consumer WTP leads to higher profits. In the following subsections, we discuss key model features.

4.1.1 Willingness-to-pay (WTP) and willingness-to-accept (WTA)

One of the ways to build a model that explains product cannibalization in CLSCs is to examine the characteristics of consumers' decision-making behavior. To this end, we focus on consumers' willingness-to-pay for given product prices.

Definition IV.1. Willingness-to-pay (WTP) is the maximum value a buyer is willing to give away to receive an item.

It is commonly assumed that consumers make purchase decisions based on their willingness-to-pay. For example, *Shioda et al.* (2011) determine the optimal prices of multiple types of products assuming that consumers make a purchase decision based on their WTPs. In their model, a consumer chooses a product that maximizes his/her utility and purchases the product if the maximum utility is positive. Here the utility of a consumer is defined as the surplus from a transaction, i.e., $\theta - p$, where θ is the consumer's WTP and p is the product price.

Our model builds on a similar consumer choice mechanism. Consumers have the option to buy a new product or a remanufactured product. Each consumer will choose the one that maximizes his/her utility. We hypothesize that product cannibalization, if it exists, is related to the utility maximizing decision mechanism. On the one hand, the sales volume of new products will be reduced if consumers find that remanufactured products offer better utilities. On the other hand, companies can reach a new consumer segment with the sales of remanufactured products creating new demands. The results from our model show that the knowledge of consumers' WTP is a key to quantifying the change in the demand for each product type and determining optimal product prices.

Paralleling the concept of a buyer's WTP is a seller's willingness-to-accept.

Definition IV.2. Willingness-to-accept (WTA) is the minimum value a seller is willing to accept in exchange of an item.

WTA is addressed in *Okada* (2010). Consumers are sellers in the reverse channel transactions for trading postconsumer products. The firm offers a buyback reward to those who return their used products. The concept of WTA is needed to determine whether or not a consumer will *accept* a given buyback reward. This will be explained in §4.2.3.

4.1.2 Uncertainty and risk aversion

Risk aversion as a way to avoid potential future regrets is a natural decision-making mechanism when decision makers in a CLSC perceive uncertainties and ambiguities. This is one of examples where uncertainty in return flows complicates the decision-making in CLSCs. Firms and consumers in CLSCs face critical uncertainties and ambiguities, e.g., with regard to product quality, which generally results in risk aversion. For consumers, price and other relevant attributes such as quality are key factors for deciding whether or not to purchase the products. Consumers are generally less willing to pay for a product that has uncertain quality. This risk aversion often becomes a major barrier for the growth of a recycling market. For example, it is well known that consumers tend to avoid retreaded tires and re-refined motor oil although these are advertised and certified to be safe (*Environment Policy Committee*, 2005). In the similar manner, establishing stable and financially viable return flows is impeded by firms' lower willingness-to-pay for the uncertain residual value of postconsumer products and consumers' lower willingness-to-accept that requests a higher buyback reward (to avoid future regret in case what they returned turns out to have a higher value than the buyback reward). As such, understanding the influence of uncertainty on risk averse decision-making is fundamental for understanding product cannibalization and thus promoting the growth of CLSCs and associated markets.

Figure 4.1 shows the perceived value of an item for a risk averse customer and a risk averse seller when they see uncertainty in the quality. In the literature (*Okada*, 2010), risk aversion is characterized by a concave curve. For convenience, we represent the seller's utility profile as a convex curve as shown in Figure 4.1(b) by interpreting the vertical axis as the amount of item value transferred to the customer.

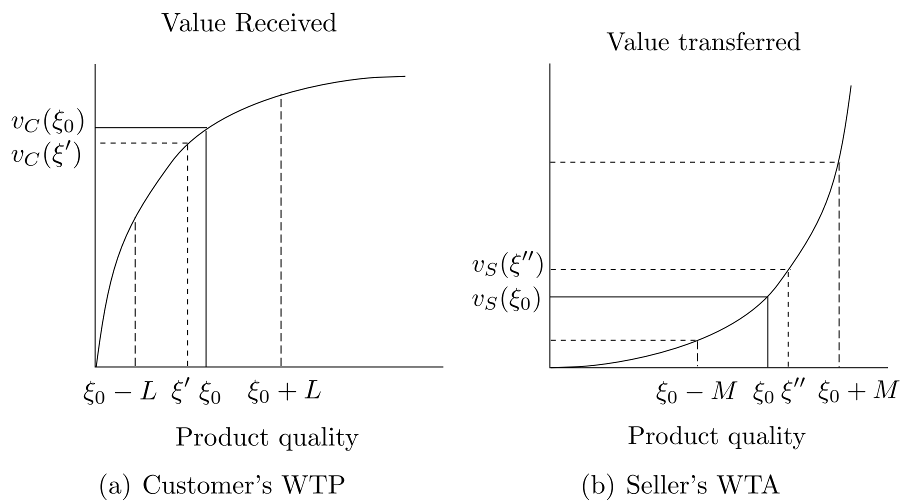


Figure 4.1: WTP vs. WTA (*Okada*, 2010)

In Figure 4.1(a), the horizontal axis represents the quality level of a product. When there is no uncertainty in the product quality, the customer recognizes the quality as ξ_0 and will purchase the product as long as he/she pays a price, say p_0, that is lower than $v_C(\xi_0)$. In this way, the customer obtains a positive surplus $v_C(\xi_0) - p_0$. Suppose that the customer is not sure about the product quality such that he/she is only able to guess that the quality would belong to a range $[\xi_0 - L, \xi_0 + L]$. In this case, the average utility becomes $v_C(\xi')$ that is lower than $v_C(\xi_0)$ due to the risk aversion represented by the concavity of the utility curve. With a decrease in the customer's WTP from $v_C(\xi_0)$ to $v_C(\xi')$, the seller may need to offer a price, say p', that is lower than p_0 in order to sell the item.

Assuming that both parties share a common belief about the quality level ξ_0, the seller knows that an item value $v_S(\xi_0)$ is transferred to the customer when there is no uncertainty in the quality. Thus, the seller will sell the item as long as it receives a reward, p_0, higher than $v_S(\xi_0)$ in order to obtain a positive surplus $p_0 -$

$v_S(\xi_0)$. However, when there exists uncertainty in product quality, the seller's WTA is adjusted such that the least acceptable reward is increased from $v_S(\xi_0)$ to $v_S(\xi'')$ due to the risk aversion. This may require a reward, p', higher than p_0 for the item to be sold.

In summary, an item is traded if

$$v_S(\xi_0) < p_0 < v_C(\xi_0) \tag{4.1}$$

when there exists no perceived uncertainty in the item value. If both of them do not know for sure the quality of an item, the item price, p', must satisfy

$$v_S(\xi'') < p' < v_C(\xi') \tag{4.2}$$

for a transaction to be established. Note that p' has a narrower range than p_0, i.e., $v_S(\xi_0) < v_S(\xi'') < p' < v_C(\xi') < v_C(\xi_0)$. Also, as the degree of risk aversion increases (i.e., the customer's and the seller's utility curves become more concave and more convex, respectively), $v_C(\xi')$ decreases and $v_S(\xi'')$ increases. This gives a smaller range for p'.

4.1.3 Related literature

In the field of traditional forward supply chains, *Moorthy and Png* (1992) investigate a market segmentation and product cannibalization problem to show that a seller may face product cannibalization with a possible profit decrease if low-end and high-end products are simultaneously introduced into a market. Although their consumer choice mechanism is similar to that in our model, the result may not be directly applicable to the case of CLSCs. This is because product cannibalization in CLSCs is an outcome of complex interactions between forward and reverse material flows, unlike that in traditional forward supply chains.

Quantitative models that address product cannibalization in CLSCs, however, have been rare. *Atasu et al.* (2008) study a remanufacturing strategy for a segmented market, where there exist "primary" consumers who have low valuation for a remanufactured product and "green" consumers who value new and remanufactured products the same. Consumers' WTP is assumed to be uniformly distributed. The size of the green market segment and primary consumers' low evaluation are identified as the main drivers for product cannibalization. From the perspective of our modeling approach, a consumer in the green market segment is not risk averse. Contrary to this, the market segmentation in our model is continuous in the sense that every consumer is risk averse and can belong to any market segmentation. It is each consumer's WTP and quality perception that determine the purchase decision and the associated market segment.

Ferguson and Toktay (2006) find conditions under which remanufacturing is profitable for OEMs. Further, they show that allowing the remanufacturing of a third-party competitor may adversely impact the OEM's profit performance. *Majumder and Groenevelt* (2001) report a similar result for a competition between an OEM and a remanufacturer. The results from our model support the conclusions of *Majumder and Groenevelt* (2001).

Mitra (2007) addresses a revenue management problem in a CLSC using a linear price-demand model, which is effectively equivalent to assuming a uniform WTP. *Debo et al.* (2005) study product remanufacturability problems using a simple hypothetical non-uniform WTP distribution. We argue that a uniform distribution of WTP is unlikely in real situations. It is not even obvious whether a WTP distribution would be unimodal or multimodal. While pricing decisions require the knowledge of the market, previous research in CLSCs has mainly focused on operational problems, with simple assumptions for market characteristics, primarily to obtain analytical solutions, e.g., "A common assumption used in almost all models is that consumers are uniformly distributed in their willingness-to-pay for a product between 0 and 1 (*Ferguson*, 2009)." Our research extends this stream of research by employing general WTP distributions such as normal distributions and Gaussian mixtures in price optimization in CLSCs. One of the objectives in this chapter is to experiment with various WTP distributions and compare the results to the uniform WTP case.

Recently, *Ovchinnikov* (2011) has addressed an optimal remanufacturing strategy considering product cannibalization. His model is based on the observation that consumers often infer quality from price, i.e., a very low price may signal inferior quality. The author tries to explain consumers' switching from new to remanufactured products by an inverted U-shape curve and compares this approach to a WTP-based pricing model, where consumers always prefer a cheaper price. As this is the most related paper to our research, we summarize the difference between our model and *Ovchinnikov* (2011) in Table 4.1.

As noted in the literature (*Voelckner*, 2006), the actual shape of a WTP distribution is crucial for determining optimal prices and estimating demands for new and remanufactured products. For example, when a market contains two distinct clusters of high valuation and low valuation consumer segments, a moderate price change may not convert some of the high valuation consumers to low valuation consumers, or vice-versa, which the inverted-U shape model may not explain. Our results show that optimal prices and demands change significantly when there exist distinct clusters of consumer segments.

In the marketing literature, estimating consumers' WTP has been an important issue for determining optimal prices and developing new products. *Breidert et al.* (2006) provide a good review of various methods for estimating consumers' WTP. In order to better focus on our research topic, we assume that information of consumers' WTP is given.

Table 4.1: A comparison of two models that address product cannibalization in CLSC.

	Ovchinnikov (2011)	Our model
profit margins	the new product is assumed to have a higher profit margin than the remanufactured product	profit margins for new and remanufactured products are determined by optimal prices
product cannibalization	product cannibalization is explicitly modeled in the objective function	product cannibalization may or may not occur depending on optimal prices and WTP distribution
decision variables	decision making only in reverse production activities: collection quantity, remanufacturing quantity, and remanufactured product price	coordinated pricing decisions in both forward and reverse production activities: new product price, remanufactured product price, and buyback price
consumer purchase	based on the inverted U-shape curve	based on product prices and quality perceptions
demand	deterministic	deterministic
solution	analytical solution	numerical solution

4.2 Monopoly

In this section, we develop a model for determining optimal prices when consumers are risk averse in purchasing remanufactured product and reselling their used products. We first consider a firm that monopolizes the market with its new and remanufactured products, and then extend it to a duopoly case in §4.3 where the firm and an external remanufacturer compete with each other.

4.2.1 The model

We consider a case where remanufactured products are made from recovering used products collected from the market through product take-back activities. In the market, consumers are risk averse and the degree of risk aversion is influenced by the amount of uncertainty they perceive in the quality of a given product. In what follows, we use the notations explained in Table 4.2.

4.2.2 WTP and WTA in forward material flows

Consider a firm that sells new and remanufactured products. Consumers are assumed to be sensitive to price and quality of the products. The firm wishes to determine: p_n = price of a new product, p_r = price of a remanufactured product, and b = buyback price of a used product. The primary source of uncertainty is the quality of remanufactured products. This is due to variation in consumers' usage patterns. Compared to this, the relative level of uncertainty in new products is negligible. To better focus on investigating the influence of uncertainty in the return flows, we make the following assumption.

Assumption IV.1. *There is no uncertainty in the quality of new products. However, consumers perceive uncertainty in remanufactured products.*

With the above assumption, the decision makers' utility profiles with regard to

Table 4.2: Notation

Notation	Meaning
p_n	price of new product (decision variable)
p_r	price of remanufactured product (decision variable)
b	buyback price (decision variable)
c_n	unit manufacturing cost
c_r	unit remanufacturing cost
p_s	unit purchase cost of raw material
θ	consumer's WTP for a new product
θ_{max}	consumer's maximum WTP for a new product
$v_C(\xi)$	consumers' degree of risk aversion about buying a remanufactured product
$v_S(\xi)$	firm's WTA in the transaction of remanufactured products
ξ	quality level of remanufactured product
δ	consumer's discount factor for future resale value of a purchased item
$f_\theta(\cdot)$	distribution of consumers' WTP for a new product
ϕ	consumer's WTA for buyback price for returning a new product
ϕ_{max}	maximum WTA for buyback price in the market
$w_C(\zeta)$	consumers' degree of risk aversion for accepting a buyback price
$w_S(\zeta)$	firm's degree of risk aversion about buying used products
ζ	quality level of used product
η	yield rate of remanufacturing process
$f_\phi(\cdot)$	distribution of consumers' WTA for buyback price

the transaction of a new product become linear because they are not risk-averse. However, we consider risk aversion in the transactions of remanufactured products, assuming appropriate shapes of utility curves for the decision makers.

Assumption IV.2. *A risk averse buyer's utility curve is concave in the quality of products. The utility in this case represents the value received. A risk averse seller's utility curve is convex in the quality of products. In this case, the utility represents the value transferred to the buyer.*

With the above assumption, the risk averse seller wishes to quote a price p_r that is higher than $v_S(\xi'')$:

$$p_r \geq v_S(\xi''), \tag{4.3}$$

where ξ'' is the perceived quality level for the remanufactured product.

As for selling remanufactured products, the degree of risk aversion of the seller would be much lower than that of buyers for the reason that the remanufacturing of used products is supposed to meet a certain quality level. In other words, the firm sells remanufactured products only when it is certain about the product quality. Therefore, we can assume that the seller sees no uncertainty in the quality of remanufactured products, i.e., $\xi_0 = \xi''$. Let c_r denote the unit remanufacturing cost. The seller invests $b+c_r$ to produce a remanufactured product. This is the minimum WTA for the seller. Thus, we simplify (4.3) as

$$p_r \geq b + c_r = v_S(\xi_0). \tag{4.4}$$

It is reasonable to assume that the degree of risk aversion in a market would be influenced by the type of product. For example, consumers tend to be more risk averse about used tires than used vehicles, i.e., the used tire market is perceived riskier by consumers than the used vehicle market. We will consider average risk aversion in a market. Average risk aversion is addressed in, for example, *Spence* (1977), *Athey* (2002), and *Duch* (2010). Let $v_C(\xi')$ denote consumers' average risk aversion for remanufactured products, where ξ' is the perceived quality level. Without loss of generality, we assume that the value of $v_C(\xi')$ is normalized such that $v_C(\xi') \in [0,1]$ for $\xi' \in [0,1]$. When $v_C(\xi') = 1$, consumers value the new and remanufactured products the same. Maximum risk aversion occurs with $v_C(\xi') = 0$ when consumers' quality perception for a remanufactured product is zero. Any value between 0 and 1 for $v_C(\xi')$ indicates an intermediate case. Although we assume the same risk aversion characteristics for the entire consumer group, each individual may have a different utility level for a given product quality. Consider a consumer whose WTP for a new product is θ. We can then scale $v_C(\xi')$ by θ and define $\theta v_C(\xi')$ as the WTP of the consumer for a remanufactured product with a quality level ξ'. Note that the relation $\theta v_C(\xi') \leq \theta$ holds to imply that WTP for a remanufactured product will not be higher than that for a new product.

We assume that the firm collects used products at a unit price b regardless of whether the collected products were previously sold as new products or as remanufactured products. Consumers use their purchased products for one time period, and have an option to resell their used products to the firm. Consumers take this into account in their purchase decision by giving a weight factor δ to the future resale value of their used products. A higher value for δ implies that consumers highly value the future resale value δb. A lower value for δ represents the case where the future resale value of the used product is not an important decision factor in the purchase decision. As such, consumers can use a future resale value as a discount factor for a product price. Considering this, we derive formulae for describing a consumer's choice between a new and a remanufactured products with risk aversion. A consumer with a WTP θ will purchase a new product if

$$\theta - p_n + \delta b \geq \theta v_C(\xi') - p_r + \delta b \tag{4.5a}$$

$$\theta - p_n + \delta b \geq 0 \tag{4.5b}$$

and will purchase a remanufactured product if

$$\theta v_C(\xi') - p_r + \delta b \geq \theta - p_n + \delta b \tag{4.6a}$$

$$\theta v_C(\xi') - p_r + \delta b \geq 0. \tag{4.6b}$$

The left-hand-side and right-hand-side of (4.5a) are the utilities the consumer obtains from buying a new product and a remanufactured product, respectively. In addition to this, (4.5b) ensures that the utility from buying a new product is nonnegative. The purchase conditions (4.6a) and (4.6b) for a remanufactured product are constructed in the similar manner.

It follows from (4.5) that the consumer will buy a new product if

$$\theta \geq \max\left\{\frac{p_n - p_r}{1 - v_C(\xi')}, p_n - \delta b\right\} \tag{4.7}$$

or, from (4.6), buy a remanufactured product if

$$\frac{p_r - \delta b}{v_C(\xi')} \leq \theta < \frac{p_n - p_r}{1 - v_C(\xi')} \tag{4.8}$$

or not buy anything if

$$\theta < \frac{p_r - \delta b}{v_C(\xi')}. \tag{4.9}$$

The minimum WTP for buying a new item expressed by (4.7) is a maximum of two quantities. The first term $(p_n - p_r)/(1 - v_C(\xi'))$ can be interpreted as an additional investment, $p_n - p_r$, for buying a new item, which is adjusted by the uncertainty effect $1/(1 - v_C(\xi'))$. The second term, $p_n - \delta b$, is the new product price perceived by the consumer. This result is intuitive because a consumer will buy a new item if he/she is willing to either (i) pay an additional amount that is needed to buy a new item or (ii) pay a higher amount than the perceived price for a new item. By (4.8), the WTP of those who buy a remanufactured item is bounded from below by the perceived price $p_r - \delta b$, which is adjusted by the uncertainty effect $v_C(\xi')$, and the WTP will not be higher than the uncertainty-adjusted additional amount $(p_n - p_r)/(1 - v_C(\xi'))$ needed for buying a new item.

As p_r becomes closer to p_n, more consumers will choose the new product because of the decreased price gap between the two types of products. In an extreme case where $p_r = p_n$, remanufactured and new products are equivalent and any incremental price difference will cause the market to shift dramatically.

Next, we characterize the upper bound for p_r for which demands for remanufactured products exist.

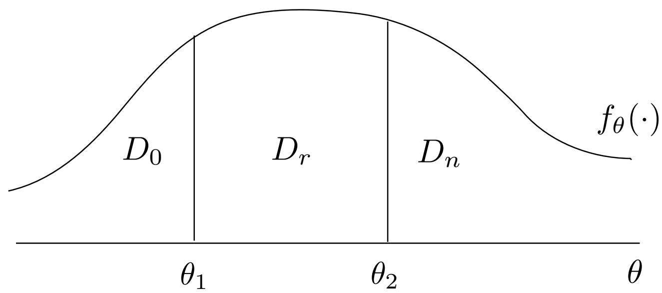
Proposition IV.1. *There exists a demand for the remanufactured product if the following inequality holds:*

$$p_r < v_C(\xi')p_n + (1 - v_C(\xi'))\delta b. \tag{4.10}$$

Proof. The upper bound for p_r is easily obtained from (4.8). We need the following to be satisfied such that (4.8) forms a non-empty range for θ:

$$\frac{p_r - \delta b}{v_C(\xi')} < \frac{p_n - p_r}{1 - v_C(\xi')}.$$

The result follows from rearranging the above inequality. Note that the right hand side of (4.10) is a convex combination of two prices p_n and δb, where $\delta b < p_n$. Thus, (4.10) is sufficient for satisfying $p_r < p_n$, which implies that the upper bound of θ in



$$\theta_1 = \frac{p_r - \delta b}{v_C(\xi')} \qquad \theta_2 = \frac{p_n - p_r}{1 - v_C(\xi')}$$

Figure 4.2: Distribution of consumers' WTP (θ) for buying a remanufactured product.

(4.8) will be positive and there exist nonnegative values for θ that satisfy (4.8). $\square$

The lower bound (4.7) for WTP of those who buy new products implies that there could exist a group of consumers who are willing to pay the amount $(p_n - p_r)/(1 - v_C(\xi'))$, but no more than the perceived price $p_n - \delta b$, i.e., $p_n - \delta b > (p_n - p_r)/(1 - v_C(\xi'))$, or equivalently, $p_r > v_C(\xi')p_n + (1 - v_C(\xi'))\delta b$. In other words, there could exist consumers who think the new product offers a better value than the remanufactured one, but do not buy anything because their WTP is not high enough to buy the new product. Proposition IV.1 shows that this does not happen when there exists a demand for the remanufactured product.

Note that θ is a unique characteristics of an individual consumer in the market. This implies that we can assume a distribution of θ as illustrated in Figure 4.2. The illustration is a generalized version of the uniformly distributed WTP discussed in *Ferguson* (2009). For a normalized market with total population size 1, the distribution denoted by f_θ represents consumers' WTP characteristics of the market. In the literature, the assumption of a uniform distribution for θ is common. We hypothesize that it will be more likely that the distribution is left-skewed, right-skewed, or symmetric in real situations depending on the degree of risk aversion, market segmentations, and the type of products. Further, there could be multiple peaks in WTP distribution in a clustered market. Our model can address such various cases by using an appropriate distribution function f_θ to fit a given situation.

The area under the function f_θ represents the amount of demand for new products (D_n), demand for remanufactured products (D_r), and the number of no purchases (D_0). The boundary between two regions D_0 and D_r, denoted by θ_1, is determined by the perceived price level for the remanufactured product. Consumers with a higher WTP than this will purchase the remanufactured product. On the other hand, the boundary between D_r and D_n, denoted by θ_2, is determined not by the price level for the new product, but by the price difference between the new and the remanufactured products. This implies that those who consider buying the remanufactured product will switch to the new product if they can afford the additional amount that is needed for buying the new product. As the uncertainty in product quality increases, the degree of risk aversion of consumers increases and consumers become less willing to pay for a remanufactured product, i.e., the value of $v_C(\xi')$ decreases. This leads to higher demands for new products, lower demands for remanufactured products, and a higher number of no purchases. The opposite happens when there is less uncertainty in product quality.

The value of θ_1 determines the amount of total demand, and the value of θ_2 determines the level of product cannibalization of new product by remanufactured product. An increase in θ_2 indicates that demand for new product is cannibalized by demand for remanufactured product. As such, it would be interesting to see when the firm can create new demands while minimizing product cannibalization, i.e., decreases in θ_1 and θ_2. We address this question via numerical experiments.

4.2.3 WTP and WTA in reverse material flows

The decision makers' roles change in the reverse channel, i.e., the firm is the buyer and consumers are sellers. But it is the firm who still determines the price for the transaction. Let ζ, $0 \leq \zeta \leq 1$, denote the quality of a used product. Consider a consumer who transfers utility ϕ to the buyer when the quality of the traded product is as new, i.e., $\zeta = 1$. Let $\phi w_C(\zeta)$ denote the utility the customer transfers to the buyer when the quality level of the used product is less than new. Similar to Figure 4.1(b), $w_C(\zeta)$ is convex in ζ and $w_C(\zeta) \in [0, 1]$. The value of the transferred product will be recognized as $\phi w_C(\zeta_0)$ if the quality level ζ_0 of the used product is perceived with certainty. With uncertainty, it becomes $\phi w_C(\zeta')$ which is higher than $\phi w_C(\zeta_0)$ due to the risk aversion of the consumer. Note that it is the consumer who 'sells' items (i.e., used products) in the reverse channel. Thus, we assume the same risk aversion mechanism that we applied to the firm in the forward channel. This is depicted by Figure 4.1(b). Intuitively, sellers will quote a higher price when they see uncertainty, whether they are engaged in forward or reverse transactions.

The consumer requests a higher buyback price b with an increase in quality uncertainty of the used product. In order to collect the consumer's used product, the firm has to offer a buyback price that satisfies $b \geq \phi w_C(\zeta')$. This condition specifies who will return a used product for a given buyback price b, i.e.,

$$\phi \leq \frac{b}{w_C(\zeta')}. \tag{4.11}$$

This implies that consumers with a WTA less than $b/w_C(\zeta')$ will return their used product. The more uncertain consumers are about the quality of remanufactured products, i.e., a higher value for $w_C(\zeta')$, a smaller number of them will return their used products. We assume a nonuniform distribution for ϕ as we did for θ (Figure 4.3). The amount of collected used products can be computed by integrating $f_\phi(\cdot)$ up to $b/w_C(\zeta')$. When the uncertainty in the return flows increases, $w_C(\zeta')$ increases, and thus the return volume decreases.

Lastly, we use $w_S(\zeta'')p_s$ to denote the quality level of a used product as evaluated

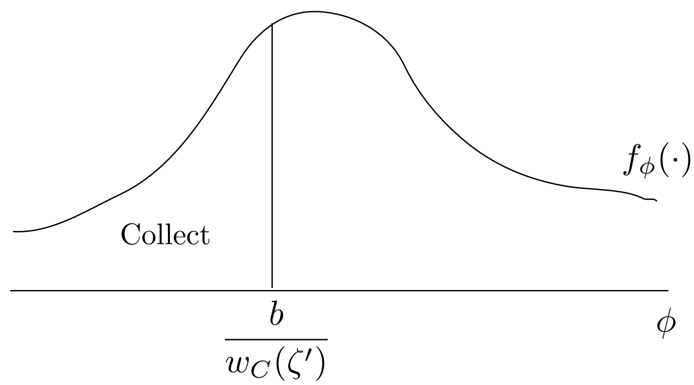


Figure 4.3: Distribution of consumers' WTA (ϕ) for a buyback price

by the firm. This is a concave function in ζ'', $0 \leq \zeta'' \leq 1$, such that $w_S(\zeta'') \in [0, 1]$. The maximum quality level $w_S(\zeta'')p_s$ is p_s when $\zeta'' = 1$, which is equivalent to buying a new material at price p_s. Uncertainty in the quality of a used product reduces the firm's WTP, i.e., $w_S(\zeta'') \leq w_S(\zeta_0)$. Thus, the buyback price will be determined at a lower level when there exists uncertainty in the quality of collected products, i.e.,

$$b \leq w_S(\zeta'')p_s. \tag{4.12}$$

Combining (4.4) and (4.12), we obtain $b \leq \min\{p_r - c_r, w_S(\zeta'')p_s\}$. The firm has an incentive to collect used product only if this condition is satisfied. This upper bound for b is influenced by the uncertainty in the quality of used products and the degree of risk aversion of the firm. For example, a higher uncertainty in product quality, thus a lower value for $w_S(\zeta'')$, may decrease the upper bound for b. Note that it is in the firm's best interest to offer as small a buyback price b as possible, which is just sufficient to induce consumers to return their used products. Any higher buyback price would only incur unnecessary costs. Therefore, we expect that the upper bound (4.12) will not be tight for an optimal buyback price b. This implies that the knowledge of the respective distributions of WTP (in forward channel) and WTA (in reverse channel) of consumers is more critical than that of the firm's.

4.2.4 Determination of optimal prices: p_n, p_r, and b

We are now ready to formulate a model that can be used to determine optimal prices for new, remanufactured, and used products considering quality uncertainties and consumers' WTP, θ with distribution $f_\theta(\cdot)$, and WTA, ϕ with distribution $f_\phi(\cdot)$. The firm's objective is to maximize the profit function $\Pi(p_n, p_r, b)$:

$$\begin{aligned}\Pi(p_n, p_r, b) = (p_n - c_n - p_s) \int_{\frac{p_n - p_r}{1 - v_C(\xi')}}^{\theta_{max}} f_\theta(z) dz \\ + (p_r - c_r) \int_{(p_r - \delta b)/v_C(\xi')}^{\frac{p_n - p_r}{1 - v_C(\xi')}} f_\theta(z) dz \\ - b \int_{\phi_{min}}^{b/w_C(\zeta')} f_\phi(y) dy \end{aligned} \tag{4.13}$$

subject to the following constraints:

$$\int_{(p_r - \delta b)/v_C(\xi')}^{\frac{p_n - p_r}{1 - v_C(\xi')}} f_\theta(z) dz \leq \eta \int_{\phi_{min}}^{b/w_C(\zeta')} f_\phi(y) dy \tag{4.14a}$$

$$\phi_{min} \leq b \leq \min\{p_r - c_r, w_S(\zeta'') p_s\} \tag{4.14b}$$

$$p_r < v_C(\xi') p_n + (1 - v_C(\xi'))\delta b \tag{4.14c}$$

$$p_n < \theta_{max} \tag{4.14d}$$

In (4.13), the first term is the profit from selling new products, the second term is the profit from selling remanufactured products, and the third term is the cost of acquiring used products. The constraint (4.14a) limits the total number of remanufactured products by a fraction η of total return volume, where η is the yield rate of product recovery. The constraint on the buyback price (4.14b) comes from (4.4) and (4.12). By Proposition IV.1, there will be no return flow if (4.14c) is violated. In this case, the problem reduces to that of non-remanufacturing case, which we call the *open-loop system*. Finally, the maximum possible price for the new product is limited by consumer's maximum WTP (4.14d).

In §4.1.2, we discussed a range for an item price when decision makers are risk averse. The buyer's risk aversion determines an upper bound and the seller's risk aversion determines a lower bound for the item price. As for the buyback price b, the lower bound is given by (4.11) and the upper bound by (4.12). As for the remanufactured product price p_r, the lower and upper bounds are obtained from (4.9) and (4.10), respectively. It is easy to see that these are addressed in the constraints set (4.14). As such, the feasible region formed by (4.14) becomes smaller when the decision makers increase their risk aversion.

The model is highly nonlinear and an analytical solution is intractable. We solve the model numerically using the variable neighborhood search method (*Mladenovic and Hansen*, 1997).

4.2.5 Numerical experiments

We perform extensive numerical experiments on the model detailed by (4.13) and (4.14) to obtain useful insights about profitable product remanufacturing when consumers are risk averse in the presence of uncertainty. To this end, we define the parameter values as follows. Without loss of generality, we set $\theta_{min} = 0$ and $\theta_{max} = 10$. This defines the support of the distribution of WTP as $[0, 10]$. The optimal prices p_n, p_r, and b will take values in $[0, 10]$.[1] Similarly, we assign 0 to ϕ_{min} and 10 to ϕ_{max}. Our primary interest lies in understanding the influence of the degree of risk aversion of consumers. Thus, we use a wide range of values for $v_C(\xi')$ and $w_C(\zeta')$. We also set $\delta = 0.1, 0.3, 0.5, 0.7, 0.9$ in order to investigate the influence of consumers' weight factor for future resale value. With a higher value for δ, consumers are more interested in the future resale value of their used product and consider this in their purchase decisions. We set the values of the other parameters at reasonable values as shown in Table 4.3.

[1] For any given value to θ_{max}, the values of the other parameters and decision variables can be appropriately scaled.

Table 4.3: Parameter values for numerical experiment.

Parameters	Values
ϕ_{max}	10
ϕ_{min}	0
θ_{max}	10
θ_{min}	0
η	1
c_n	1
c_r	1
p_s	1, 2
$v_C(\xi')$	0.5, 0.6, 0.7, 0.8, 0.9
$w_C(\zeta')$	0.1, 0.2, 0.3, 0.4, 0.5
δ	0.1, 0.3, 0.5, 0.7, 0.9
$w_S(\zeta'')$	0.8

Different market characteristics are modeled by four different types of WTP distributions $f_\theta(\cdot)$ as shown in Table 4.4. We use Gaussian mixture models to account for two different clusters of consumers, i.e., low valuation and high valuation consumers. A Gaussian mixture model is flexible in the sense that it can fit a wide range of distributions. It is one of methods for representing market segmentations (*Huang et al.*, 2007). We assume a normal distribution $f_\phi(\cdot)$ with regard to consumers' WTA for buyback price as there will exist only one category of product, i.e., used products. There are a total of 36,000 cases.

Table 4.4: Distributions examined for WTP and WTA. Three cases of different standard deviation, $\sigma = 0.7$, 1.0, and 1.3, apply to all except the uniform distribution. All distributions are truncated on the finite support.

Distributions	Types
$f_\theta(\cdot)$	Normal ($\mu = 6$, 7, 8)
	Gaussian mixture ($\mu_1 = 4, \mu_2 = 7$, 30% of population at mode 1)
	Gaussian mixture ($\mu_1 = 4, \mu_2 = 7$, 70% of population at mode 1)
	Uniform $\sim [\theta_{min}, \theta_{max}]$
$f_\phi(\cdot)$	Normal ($\mu = 1$, 2, 3)

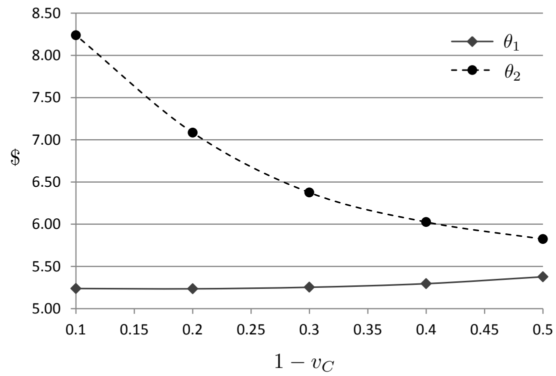
We implemented the variable neighborhood search method in C programming language to solve the model. The total required computation time for all 36,000 cases was 20 minutes on a quad-core 2.4 GHz PC. In the next section, we discuss averaged solution results for the four different WTP distributions followed by a comparison of the four different cases.

4.2.5.1 The influence of consumers' risk aversion in buying remanufactured product

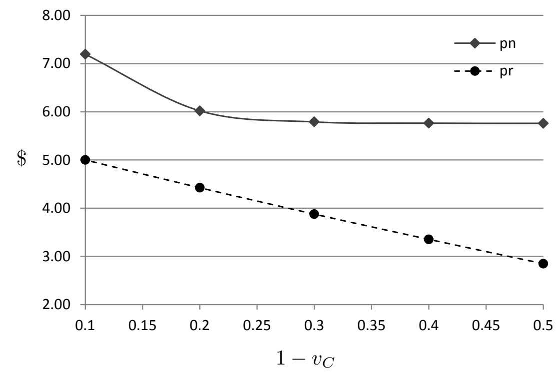
The solutions obtained for the numerical experiments provide insights that are consistent with intuition. Some of the numerical results are shown in Figure 4.4.

Consider the degree of risk aversion of consumers, which can be expressed by $1 - v_C(\xi')$, in their purchase of remanufactured products. The value of $1 - v_C(\xi')$ increases as consumers become more risk averse. Demand for remanufactured products is directly influenced by consumers' risk averse decision-making. As consumers are less certain about the quality of remanufactured products, more consumers switch to buying a new product as shown by the decreasing θ_2 in Figure 4.4(a). Also, when this happens the value of θ_1 slightly increases, which implies that more consumers will choose to not buy anything when they see more uncertainty in remanufactured products. Thus, the gain in total demand decreases (Figure 4.4(e)).

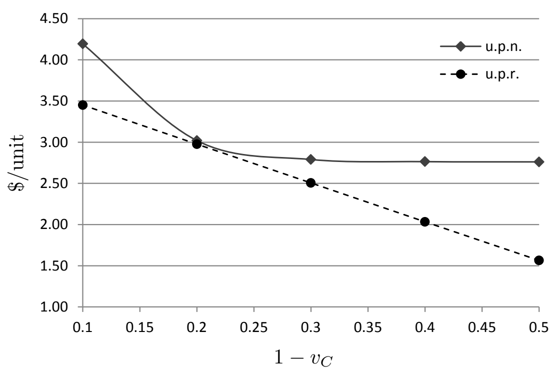
Intuitively, the firm will have to decrease the price of remanufactured product as consumers become more reluctant to buy remanufactured products (Figure 4.4(b)). As a result, the firm has a reduced unit profit for each sale of remanufactured products (Figure 4.4(c)). However, the unit profit from new products is less influenced by consumers' risk aversion on remanufactured products except the case where consumers show very low risk aversion. In this case, most of the demand for new product is cannibalized by that of remanufactured product. The optimal solution implies that the firm may implement a high price for a new product to drive a higher profit margin.



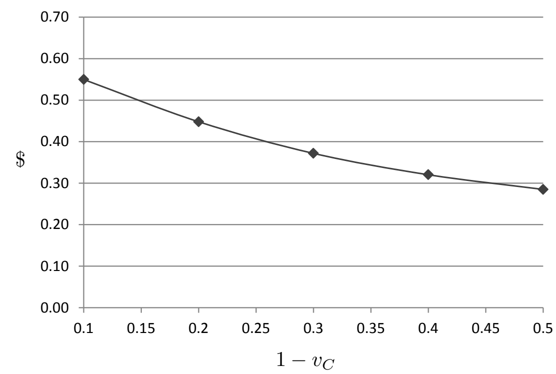
(a) θ_1 and θ_2



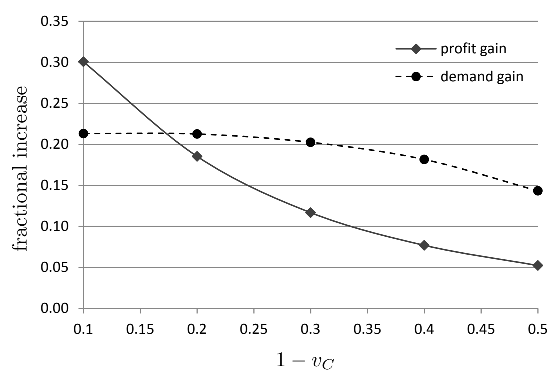
(b) p_n and p_r



(c) Unit profits (u.p.n. = unit profit for new product, u.p.r. = unit profit for remanufactured product)



(d) Buyback price



(e) Fractional increases in profit and demand from remanufacturing relative to the open-loop system

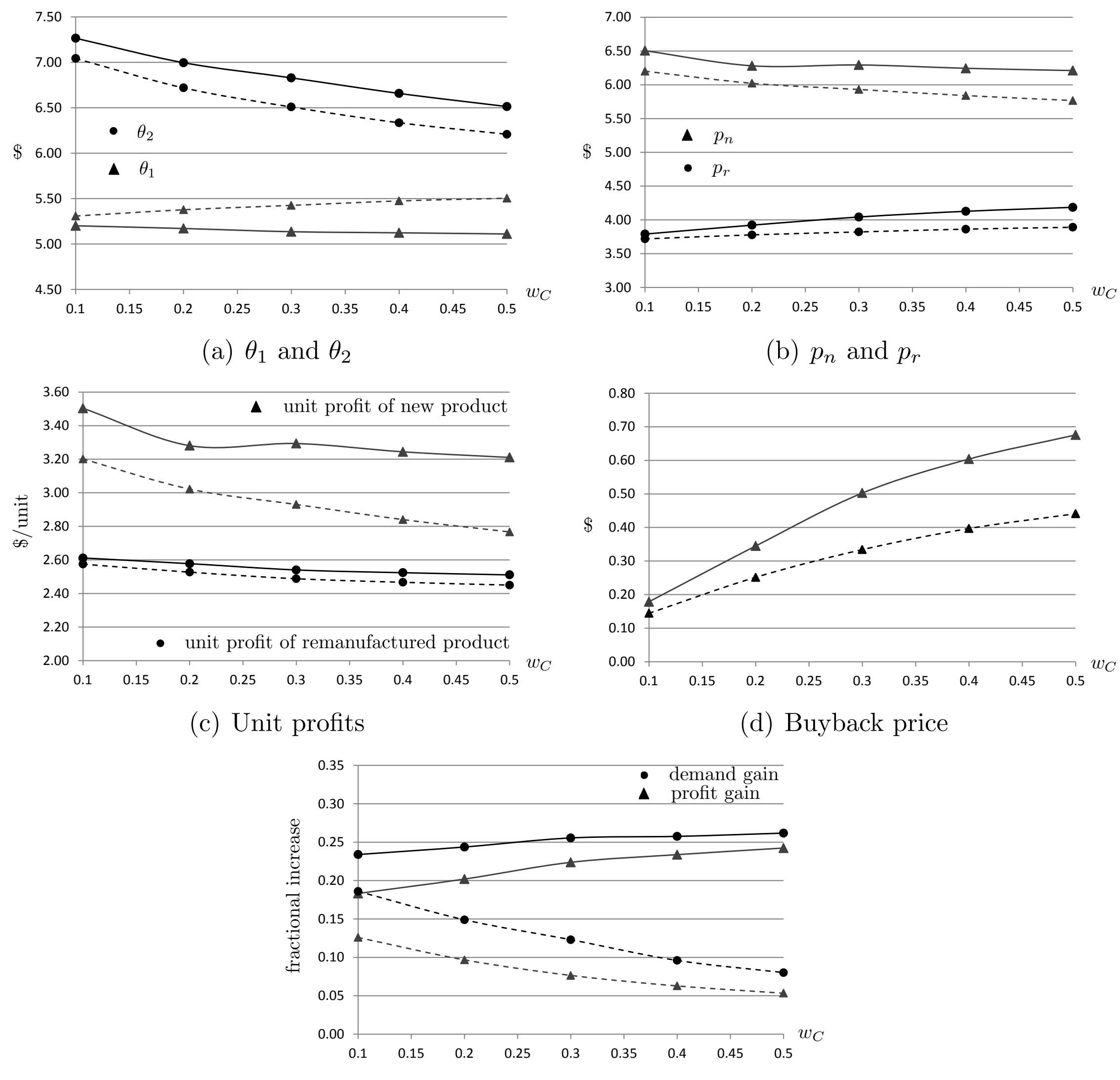
Figure 4.4: The influence of consumers' risk aversion in buying remanufactured product.

Figure 4.4(d) shows that buyback price decreases as consumers are more risk averse to buying remanufactured products. This occurs because the firm will have less incentive for collecting used products when demand for the remanufactured product rapidly decreases. In summary, a larger portion of consumers switch to the new product as consumers become more risk averse to purchasing a remanufactured product. As a result, the firm's profit decreases.

4.2.5.2 The influence of consumers' risk aversion in accepting the buyback price

In the reverse channel, consumers are risk averse in their decisions to accept a buyback reward for returning their used products. That is, they may request a higher reward when there exists uncertainty in the quality of used products. This is to avoid any potential future regret for returning a used product that is worth more than the accepted buyback price. The value of $w_C(\zeta')$ represents the degree of risk aversion in accepting a given buyback price. The buyback price also affects the transaction in the forward channel through a discounting effect on prices because consumers can anticipate a future money back reward. This discounting effect varies from one product category to another. For example, many consumers take into account the future resale value when they consider buying a car. But it is unlikely that they will also consider the future resale value of new tires. This is addressed via different values for δ in our model.

In Figure 4.5, we compare two cases when $\delta = 0.1$ and $\delta = 0.9$. Let us first consider the case where consumers give a small weight factor, $\delta = 0.1$, to the future resale value of used products. From Figure 4.2 we know that θ_1 increases with a smaller value of δ, which implies that more consumers choose to not buy anything and the total demand decreases. The firm needs to offer a higher buyback price when consumers become increasingly uncertain about the quality of their used products



(a) θ_1 and θ_2

(b) p_n and p_r

(c) Unit profits

(d) Buyback price

(e) Fractional increases in profit and demand from remanufacturing relative to the open-loop system

Figure 4.5: The influence of consumers' risk aversion in accepting a buyback price (dotted line for $\delta = 0.1$, solid line for $\delta = 0.9$).

as shown in Figure 4.5(d). The same argument applies to the case of $\delta = 0.9$. Any increase in b would enlarge the total demand by decreasing the value of θ_1. However, this change in buyback price does not contribute much to the change in the value of θ_1 because the value of the scale factor δ is small. With an increase in buyback price the firm has a tighter profit margin in reverse channel transactions. Thus, the price of remanufactured product p_r increases. As opposed to this, p_n is decreasing as a function of consumers' risk aversion because the firm expands the demand for new products and as a result it has more room to adjust the price of new products. Accordingly, the uncertainty in reverse channel reduces the price difference $p_n - p_r$ and the value of θ_2. This drives more people to prefer new products over remanufactured products. The firm has little incentive for increasing return flows by offering a higher buyback price, as the demand for remanufactured product will be small. Nevertheless, the firm has to increase the buyback price in order to acquire return flow from consumers who are reluctant to return their used products. In all, Figure 4.5(c) shows that the unit profit for each of the new and remanufactured products decreases with increasing consumers' risk aversion in the reverse channel.

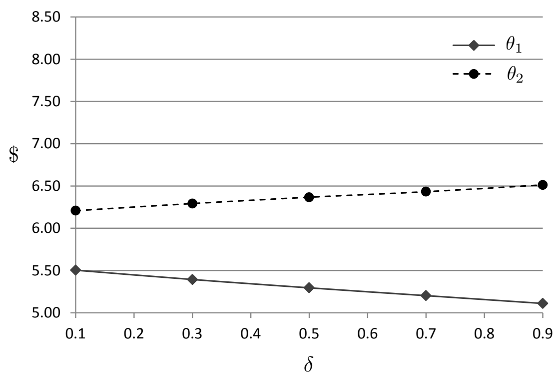
Next, we consider the case $\delta = 0.9$. Examples here include cases where products are leased to customers for a finite period of time. In this case, the price of leasing an item is equivalent to $p_n - \delta b$ in our model. The firm still has to increase the buyback price when δ is larger and consumers become more risk averse in reverse channel transactions. However, unlike the previous case for $\delta = 0.1$, the increase in buyback price is amplified by the larger scale factor $\delta = 0.9$. When the value of δ is large, an increase in the buyback price strengthens the consumers' interest in the remanufactured product. As a result, the value of θ_1 slightly decreases (Figure 4.5(a)) and the firm can increase total demand and profit when the uncertainty in the reverse channel becomes larger. This result shows that active participation of consumers is important for viable operation of the closed-loop supply chain. It may be intuitively obvious that less uncertainty in any part of the system is more desirable for improving the system performance. Interestingly, some types of uncertainty could stimulate decision makers in such a way that their decisions drive a better system performance under specific situations. When consumers are very willing to return their used products, the adverse effect of uncertainty in the reverse channel can be overcome.

4.2.5.3 The influence of consumers' weight factor for future resale value

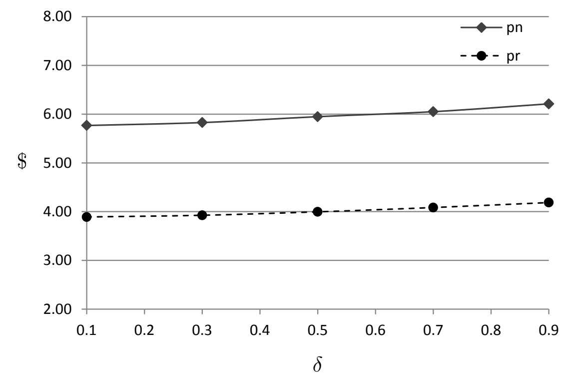
We further explore the importance of the value of δ for profitable remanufacturing. As consumers give a higher weight factor for future resale value of their used products, the firm can create new demands with a decrease in the value of θ_1. This requires more return flows from the market. Thus, the firm needs to increase the buyback price as shown in Figure 4.6(d). This is desirable for the firm as it reinforces the decrease in the value of θ_1 creating more new demand (Figure 4.6(a)). Increasing demand for the remanufactured product also cannibalizes the new product sales. Overall, as consumers are more willing to return their used products, the firm's profit increases almost linearly.

4.2.5.4 Comparison of various WTP distributions

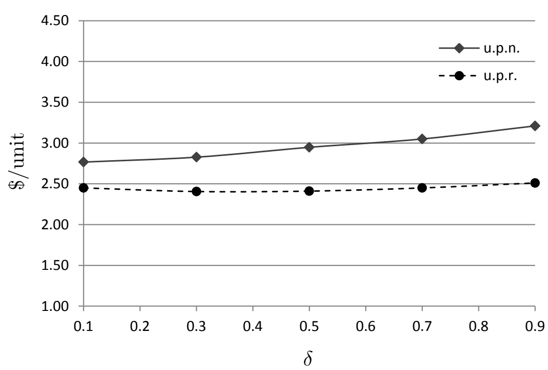
We have thus far discussed common qualitative characteristics of four different WTP distributions defined in Table 4.4. The results imply that the assumption of uniformly distributed WTP is suitable for explaining overall *qualitative* changes in optimal prices, profit, and demand with regard to the changes in the degree of consumers' risk aversion in forward and reverse channels. This shows the usefulness of the uniform WTP distribution as an analytical tool for driving qualitative insights and intuition. However, the profit/demand gains and the optimal prices can be significantly different from one distribution to another.



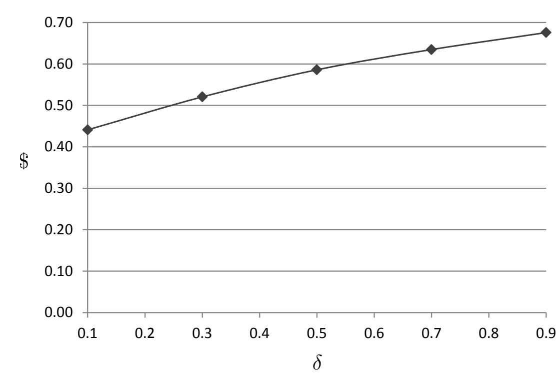
(a) θ_1 and θ_2



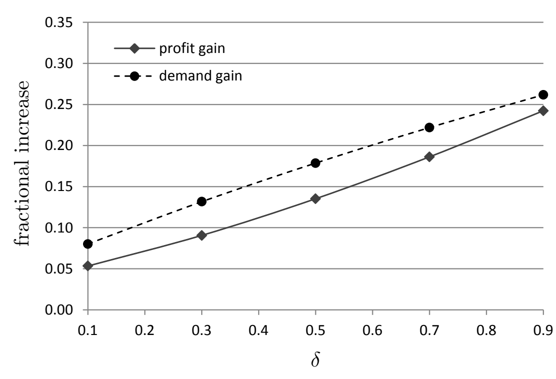
(b) p_n and p_r



(c) Unit profits (u.p.n. = unit profit for new product, u.p.r. = unit profit for remanufactured product)

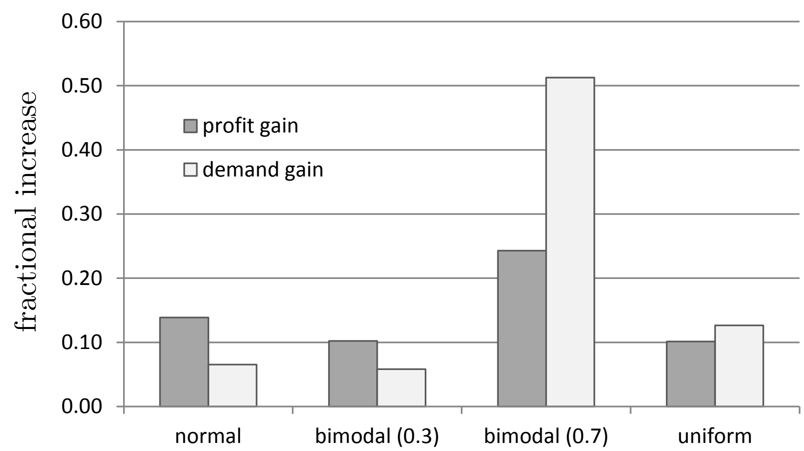


(d) Buyback price

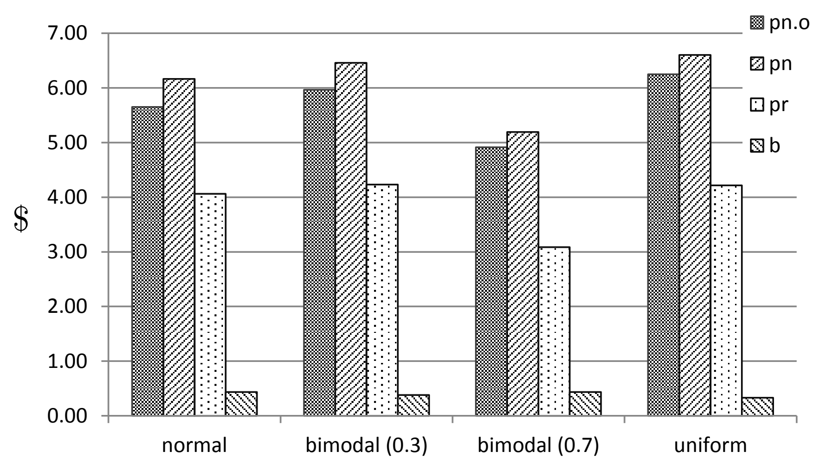


(e) Fractional increases in profit and demand from remanufacturing relative to the open-loop system

Figure 4.6: The influence of consumers' weight factor (δ) for future resale value of used products.



(a) Fractional increases in profit and demand from remanufacturing relative to the open-loop system



(b) Average optimal prices. (pn.o = price of new product in open-loop system)

Figure 4.7: Profit/demand gains and optimal prices for the four distributions defined in Table 4.4.

We compare four different WTP distributions in Figure 4.7. In bimodal (0.3) and bimodal (0.7) distributions, 30% and 70% of the population belong to the cluster with mean value at 4, respectively. Figure 4.7(a) shows that the relative profit and demand gain from remanufacturing (when compared to the open-loop case) is the highest in bimodal (0.7) case. In particular, the demand gain in bimodal (0.7) significantly outperforms all the other cases. This is reasonable given that in bimodal (0.7) market, remanufacturing can capture the demand from the large low valuation consumer cluster, which the open-loop system cannot reach unless it significantly decreases the new product price.

Optimal prices in different markets are compared in Figure 4.7(b). One can observe that the uniform WTP produces the highest optimal price for a new product whether or not the firm is engaged in remanufacturing. This is because the uniform WTP distribution overestimates the number of the high valuation consumers. Note that, according to Table 4.4, the mean of the uniform WTP distribution is $(\theta_{max} - \theta_{min})/2 = (\theta_{max} - 0)/2 = \theta_{max}/2 = 10/2 = 5$. The mean values of the other distributions are higher than 5. It might seem logical to expect a low optimal price for a WTP distribution that has a lower mean level. The numerical result shows an opposite result in Figure 4.7(b) where the uniform WTP distribution produces the highest optimal prices. This can be explained by the shape of the uniform distribution that 'overestimates' the number of high-valuation consumers.

4.2.5.5 Product cannibalization

Remanufacturing brings a profit increase to each of the four different cases. But it also accompanies product cannibalization. Figure 4.8 shows that about 30% of the demand for a new product switches to a remanufactured product in the numerical experiments. It is generally recognized that product cannibalization is an undesirable phenomenon that needs to be avoided. However, the numerical results show that

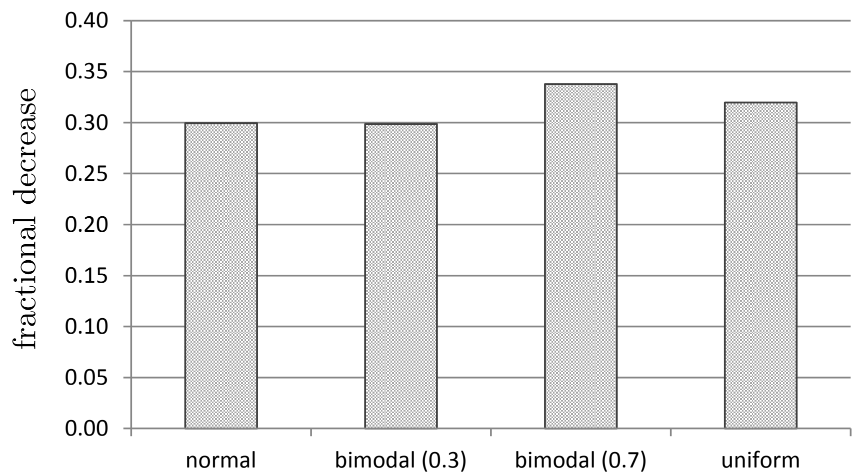


Figure 4.8: Product cannibalization (fractional decrease in new product sales)

product cannibalization is inevitable if firms wish to pursue a higher profit from remanufacturing. In fact, the issue of product cannibalization should be approached from a broader perspective, i.e., the total profit of the system.

One of the contributions of this research is that we quantify profit gains and product cannibalization considering characteristic distributions of consumers' WTP. Our model consistently demonstrates that any profit gain from remanufacturing is accompanied by product cannibalization of new product by remanufactured product but that this does not necessarily decrease overall profit. We argue that this result must be understood within the context of revenue management, i.e., firms can increase their profits by offering several different category of products with different value propositions to customers who have different levels of WTP. In this way, firms can reach different market segments and achieve better profits by adapting their systems to the specific market conditions.

4.2.5.6 What if a uniform WTP approximation is wrong?

As we mentioned earlier in this chapter, a uniform assumption for a distribution of consumers' WTP has been widely accepted in the related research field. However, there may be cases where the uniform assumption of WTP fails to capture important market characteristics. We ask the following questions: "What would be the quality of a solution in terms of profit performance when we approximate a real WTP distribution with a uniform distribution?" In other words, "What if a uniform WTP approximation is wrong?" Our model is useful to answer this question as the model can accommodate general distributions for WTP. To this end, we assume that normal, bimodal (0.3), and bimodal (0.7) WTP distributions represent real situations. There are 36,000 cases in each of three different types of WTP distributions. We then approximate each real WTP distribution with a uniform WTP distribution to obtain optimal prices, p_n, p_r, and b, that maximize the profit. For a given 'real' WTP distribution:

1. The true optimal profit, say Π^*, is obtained using the real WTP distribution.
2. The approximate profit, say $\tilde{\Pi}$, is obtained using a uniform WTP distribution.

The ratio $\tilde{\Pi}/\Pi^*$ is then defined as the solution quality of a uniform WTP approximation for the given real WTP distribution. From this numerical experiment, we identify two issues with the uniform WTP approximation.

First, we find that only 52%, 30%, and 62% of 36,000 approximate solutions are feasible for normal, bimodal (0.3), and bimodal (0.7) cases, respectively. In fact, a uniform approximation does not guarantee a solution that is also feasible in a "real" situation. This is because the feasible region defined by (4.14) depends on the WTP distribution $f_\theta(\cdot)$. Thus, a solution obtained from a uniformly approximated WTP distribution may not be directly implemented in a real market due to the feasibility issues.

Second, Figure 4.9 shows the average solution quality in the feasible cases. The uniform WTP approximation works best for the bimodal (0.3) case. It achieves 80% of optimality. In this case, the uniform distribution's overestimation of the high valuation consumer population becomes less of an issue. For the normal WTP distri-

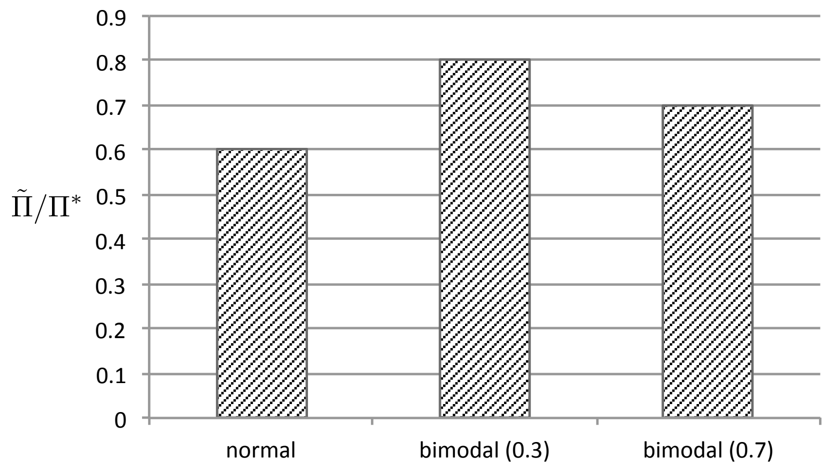


Figure 4.9: Average of solution quality, $\tilde{\Pi}/\Pi^*$, of uniform WTP approximations. ($\tilde{\Pi}$ = approximated profit via uniform WTP, Π^* = optimal profit)

bution, it achieves only 60% of optimality. Note that the uniform WTP distribution has its mean value at $\theta = 5$. Among the three types of 'real' WTP distributions, the bimodal (0.7) distribution has the smallest mean value that is closer to 5 due to its large low valuation consumer cluster centered at $\theta = 4$. Yet the solution quality for this case is outperformed by the bimodal (0.3) distribution which has a higher mean level with a large high valuation consumer cluster centered at $\theta = 7$. The implication is that approximating a real WTP distribution with a uniform distribution that has the same or similar mean value is not sufficient for obtaining a high quality solution.

Overall, the numerical result implies that companies may underperform with the uniformly approximated solution in a real market even when there exists a large high valuation consumer cluster. From the perspective of financial viability of CLSCs, it is an important issue because the profit underperformance caused by suboptimal prices may signal wrong impressions that product remanufacturing is not profitable enough when they can greatly improve the profit performance by incorporating correct market information into the price optimization process. In summary, the results demonstrate that knowledge of specific market characteristics, i.e., the distribution of consumers' WTP, is critical for determining optimal prices and the resulting profit performance for product remanufacturing operations.

4.3 Duopoly

There are many cases where remanufacturing firms compete with OEMs who do not remanufacture their postconsumer products. One of the relevant examples is the competition between OEM inkjet printer manufacturers and third party printer cartridge refillers. OEM inkjet printer manufacturers generally sell inkjet printers at very low prices, sometimes even below the actual production cost. Selling new inkjet cartridge at high prices is the major source of profit. The existence of a third-party remanufacturer who supplies cheaper alternative products could disrupt the OEM's business model. The duopoly model we develop in this section considers this type of competition between two firms when consumers are risk averse in purchasing remanufactured products.

4.3.1 The model

We consider a duopoly where an OEM sells new products and a competitor sells remanufactured products made from used products acquired from the OEM's market. The OEM determines the price p_n of the new product. The competitor determines the buyback price b and the price p_r of the remanufactured product. The duopoly model is built upon the same formulae (4.5) and (4.6), but there are now two decentralized decision makers. We model this problem as a Stackelberg game where the OEM moves first to set new product price p_n and the competitor follows in setting the buyback price b and the remanufactured product price p_r.

The OEM solves the following problem:

$$\Pi_n(p_n) = \max \quad (p_n - c_n - p_s) \int_{\frac{p_n - p_r}{1 - v_C(\xi')}}^{\theta_{max}} f_\theta(z) dz \tag{4.15a}$$

$$\text{s.t.} \quad p_n < \theta_{max} \tag{4.15b}$$

The competitor solves the following problem:

$$\Pi_r(p_r, b) = \max \quad (p_r - c_r) \int_{(p_r - \delta b)/v_C(\xi')}^{\frac{p_n - p_r}{1 - v_C(\xi')}} f_\theta(z) dz - b \int_{\phi_{min}}^{b/w_C(\zeta')} f_\phi(y) dy \tag{4.16a}$$

$$\text{s.t.} \quad \int_{(p_r - \delta b)/v_C(\xi')}^{\frac{p_n - p_r}{1 - v_C(\xi')}} f_\theta(z) dz \le \eta \int_{\phi_{min}}^{b/w_C(\zeta')} f_\phi(y) dy \tag{4.16b}$$

$$\phi_{min} \le b \le \min\{p_r - c_r, w_S(\zeta'')p_s\} \tag{4.16c}$$

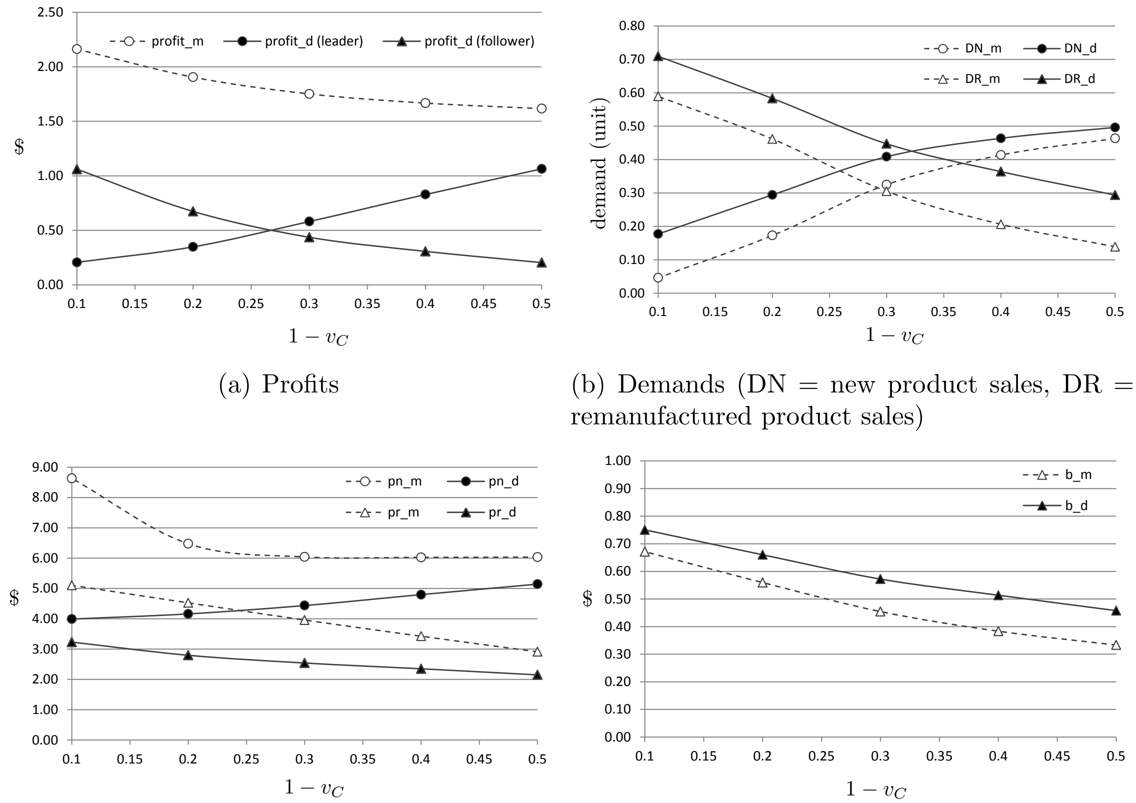
$$p_r < v_C(\xi')p_n + (1 - v_C(\xi'))\delta b \tag{4.16d}$$

The optimal solution to the Stackelberg game is generally solved by backward-induction. First, the competitor's best response (p_r, b) to a given p_n is computed. This is equivalent to expressing p_r and b as functions of p_n. Next, the leader maximizes Π_n using the information $p_r(p_n)$ and $b(p_n)$. Then, leader's decisions are replaced in the competitor's best response functions to find an equilibrium solution. An analytical solution is intractable due to the general distribution functions $f_\theta(\cdot)$ and $f_\phi(\cdot)$. Instead, we take a numerical solution approach as described in the following subsection.

4.3.2 Numerical experiments

In this subsection we conduct numerical experiments using the same data and WTP/WTA distributions shown in Table 4.3 and 4.4. The numerical computation of the Stackelberg equilibria p_n^*, p_r^*, and b^* follows the following iterative steps:

1. Choose a value for p_n from a predefined discretized range,
2. solve the competitor's problem to obtain the values of $p_r(p_n)$ and $b(p_n)$ that



(a) Profits

(b) Demands (DN = new product sales, DR = remanufactured product sales)

(c) Prices

(d) Buyback prices

Figure 4.10: The influence of consumers' risk aversion in buying remanufactured product. ('m': monopoly, 'd': duopoly)

maximize the profit Π_r, and

3. compute the OEM's profit Π_n taking into account the competitor's best responses $p_r(p_n)$ and $b(p_n)$.

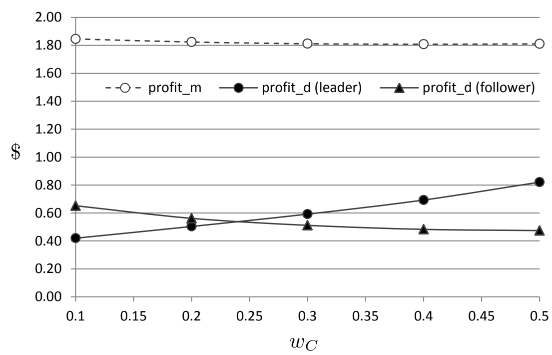
We repeat the above steps for all values of p_n and choose the one that maximizes Π_n. This also determines the equilibrium solution for the retailer, which was already computed during the iterations. A similar approach is used by *Pedroso* (1996). We use the variable neighborhood search method to maximize Π_r and Π_n in each iteration.

4.3.2.1 The influence of consumers' risk aversion on buying a remanufactured product

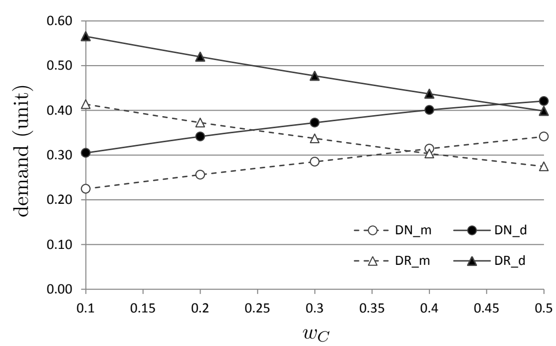
In the duopoly market, the consumers' risk aversion to the competitor's remanufactured products is an opportunity to the OEM. As consumers become increasingly risk averse in their purchase of the competitor's remanufactured product, the OEM cannibalizes the demand for the competitor's product ("DN_d" and "DR_d" in Figure 4.10(b)), increases price p_n ("pn_d" in Figure 4.10(c)), and achieves a higher profit (Figure 4.10(a)). The opposite happens to the competitor, i.e., it loses demand ("DR_d" in Figure 4.10(b)) even with decreasing price p_r ("pr_d" in Figure 4.10(c)) and profit decrease (Figure 4.10(a)). Less collection effort b will be required as there will be less demand for remanufactured product. This applies to both monopoly and duopoly situations (Figure 4.10(d)). In the monopoly, the OEM considers a tradeoff between new and remanufactured products when it determines the optimal buyback price. In the duopoly, the competitor's objective is to maximize its profit by selling a single category of product. The numerical results indicate that higher buyback price is optimal in the duopoly case. We note that the above results are obtained from numerical experiments. Thus, the validity of the results reported here applies to the specific set of numerical data used for the numerical analysis.

4.3.2.2 The influence of consumers' risk aversion in accepting buyback price

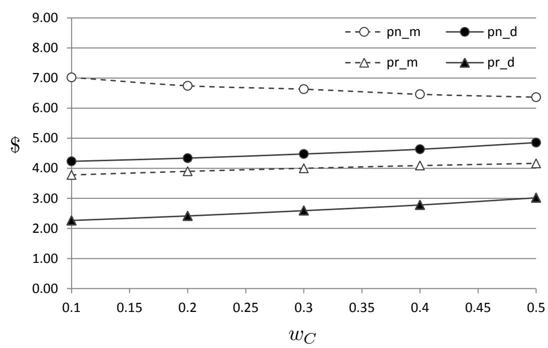
When consumers are increasingly risk averse to accepting a given buyback price to return their used products, the competitor's profit performance decreases but not as much as in the case where consumers' risk aversion to remanufactured product increases (Compare Figure 4.10(a) and Figure 4.11(a)). The risk aversion in the collection channel affects demands for remanufactured product, and the OEM takes advantage of this situation to increase its demand (Figure 4.11(b)).



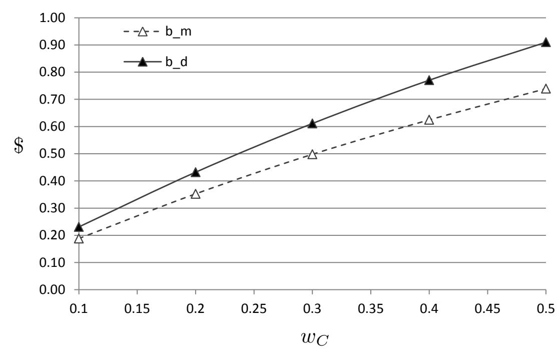
(a) Profits



(b) Demands (DN = new product sales, DR = remanufactured product sales)

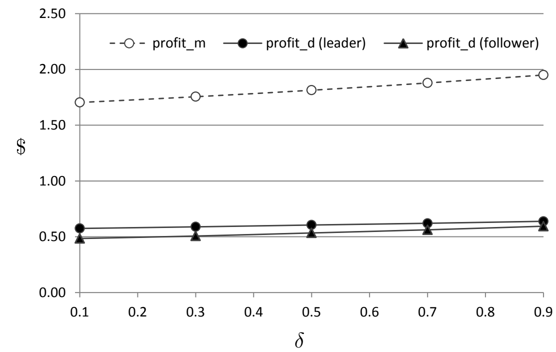


(c) Prices

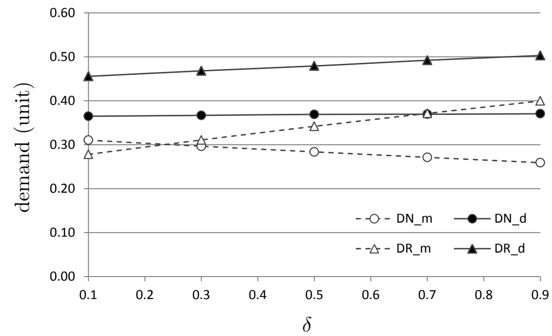


(d) Buyback prices

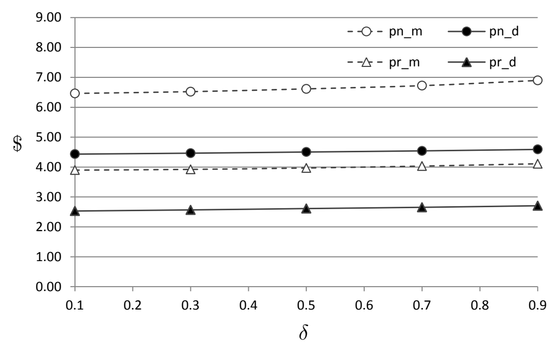
Figure 4.11: The influence of the consumers' risk aversion in accepting buyback price. ('m': monopoly, 'd': duopoly)



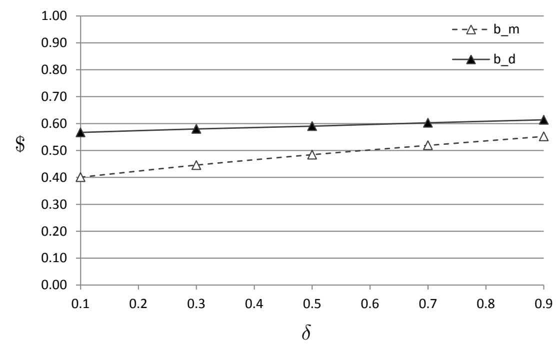
(a) Profits



(b) Demands (DN = new product sales, DR = remanufactured product sales)



(c) Prices



(d) Buyback prices

Figure 4.12: The influence of consumers' weight factor for future resale value. ('m': monopoly, 'd': duopoly)

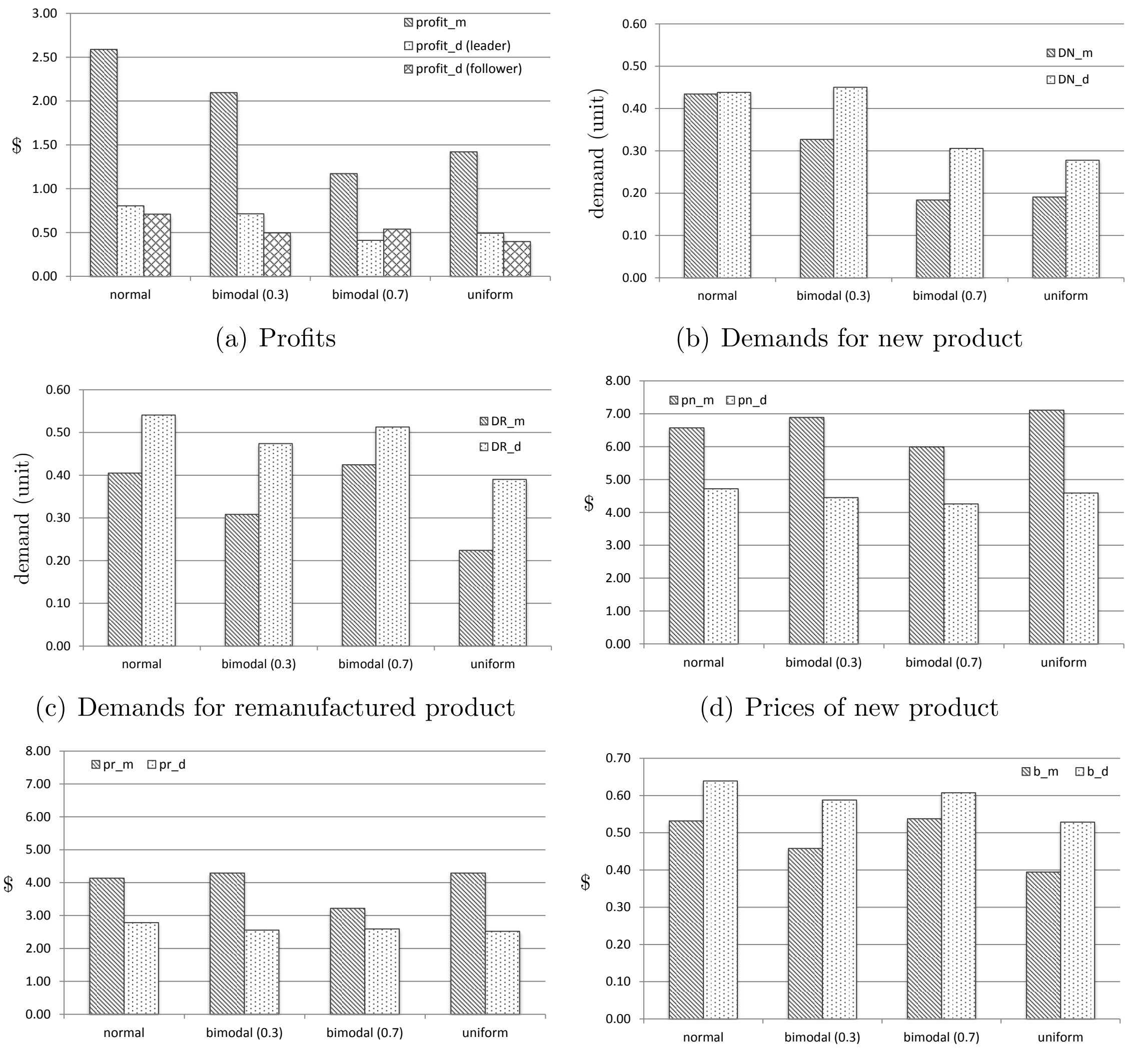
It is interesting to observe that p_n decreases in a monopoly but it increases in a duopoly with an increase in consumers' risk aversion to either purchasing remanufactured product (Figure 4.10(c)) or returning their used products (Figure 4.11(c)). In fact, this is consistent with the fact that firms can generally adopt higher prices when they monopolize the market. In other words, as consumers become more risk averse about the competitor's product, the competitor becomes weaker in the competition with the OEM. As a result, the OME's power to monopolize the market increases and it can implement a higher optimal price p_n.

4.3.2.3 The influence of consumers' weight factor for future resale value

The parameter δ can be interpreted as an attribute of the product, the market, or the entire system rather than of consumers. For example, almost all consumers trade in their used vehicle when they purchase a new vehicle. In this case, the resale value or trade-in value is one of the important factors for a purchase decision and the value of δ will be large. Unlike in the monopoly case where the OEM performs better by utilizing the opportunity from an increasing "willingness to return," δ, of used products from consumers, the two competitive firms in a duopoly do not enjoy the same benefit. In a monopoly, the OEM allows demand cannibalization of new product when consumers are highly willing to return their used products because it can increase the total profit. In a duopoly, the OEM protects its market as any demand cannibalization by the competitor may result in a profit loss. Figure 4.12(b) shows that the demand for new product maintains at almost the same level regardless of the value of δ. Consequently, the increase in demand for remanufactured product is less than that of monopoly case. The result implies that in a duopoly situation, consumers' active participation in product remanufacturing may have limited effect.

4.3.2.4 Various distributions for WTP

Finally, we compare characteristics of the competition in different markets. Figure 4.13(a) shows that the OEM's profit is significantly reduced by allowing the competitor to remanufacture used products. As the Stackelberg leader, the OEM performs generally better than the competitor. However, if the market condition is favorable for the competitor, i.e., a larger low valuation cluster as in the bimodal (0.7) case, than the competitor achieves a higher profit. The OEM gains higher demand for the new product in a duopoly. This might be desirable for those who are concerned with product cannibalization and market share. However, the expansion in the demand for new products is a result of a much lower optimal price p_n due to the competition



(a) Profits

(b) Demands for new product

(c) Demands for remanufactured product

(d) Prices of new product

(e) Prices of remanufactured product

(f) Buyback prices

Figure 4.13: Various distributions for consumers' WTP for remanufacturing product.

(Figure 4.13(d)). The competitor gains higher market share than the OEM in all four different markets (Compare Figure 4.13(b) and Figure 4.13(c)). For example, in the market where consumers' WTP is normally distributed, the OEM and the competitor's market shares are about 44% and 55%, respectively. We note that the total demand is significantly larger in duopoly than in monopoly. This is again due to lower prices, especially, the price p_r for the remanufactured product. In all, consumers benefit most from the duopoly market. They can enjoy cheaper prices in a 'greener' market where more products are collected and remanufactured.

4.4 Conclusions

In this research we investigated the influence of risk averse consumers' decisions on a firm's profit performance and product cannibalization. The contribution of this research is that we provide formal models that quantify the interactions among consumers' WTP distribution, quality uncertainty in postconsumer and remanufactured products, consumers' risk aversion, firm's risk aversion, product cannibalization, and the firm's performance.

Optimal decision-making in CLSCs, especially pricing decisions, cannot be independently solved without considering marketing activities. To this end, our approach integrates principles from Operations Research and Marketing to obtain useful insights on this previously rarely explored problem in the CLSC research area. We conducted extensive numerical experiments for monopoly and duopoly cases to obtain useful insights on the benefit of product remanufacturing. The key results are summarized in Table 4.5 and Table 4.6. In general, OEMs perform worse when consumers are more risk averse in a monopoly market. But when consumers are highly willing to participate in product returns (e.g., $\delta = 0.9$), OEMs can perform better even when consumers become more risk averse. This is because the increase in buy-back price strengthens consumers' interest in purchasing remanufactured products, which results in the creation of new demands.

Competition between an OEM and a remanufacturing firm induces higher demands both for new and remanufactured products with lower prices for new and remanufactured products. The OEM creates more demands for new products but loses the entire demand for the remanufactured products to the competitor. The numerical results show that the OEM's profit is significantly reduced by the competition with the remanufacturer. Demand for remanufactured products is cannibalized by demand for new products as consumers are increasingly risk averse about purchasing the remanufactured product. The competitor generally gains more demand than the

Table 4.5: Influence of consumers' risk aversion and 'willingness-to-return' (δ) in the monopoly case. ($D_t = D_n + D_r$, $\Delta\Pi$ = profit gain from remanufacturing)

		p_n	p_r	b	D_n	D_r	D_t	$\Delta\Pi$
Risk aversion in forward channel	↑	-	↓	↓	↑	↓	↓	↓
Risk aversion in reverse channel ($\delta = 0.1$)	↑	↓	↑	↑	↑	↓	↓	↓
Risk aversion in reverse channel ($\delta = 0.9$)	↑	-	↑	↑	↑	↓	↑	↑
Willingness to return (δ)	↑	↑	↑	↑	↓	↑	↑	↑

Table 4.6: Influence of consumers' risk aversion and 'willingness-to-return' (δ) in the duopoly case.

		p_n	p_r	b	D_n	D_r	Π_n	Π_r
Risk aversion in forward channel	↑	↑	↓	↓	↑	↓	↑	↓
Risk aversion in reverse channel	↑	↑	↑	↑	↑	↓	↑	↓
Willingness to return (δ)	↑	-	-	↑	-	↑	-	-

OEM, but with a lower profit due to its follower position in the Stackelberg game. If there exists a large low valuation cluster in the market, then the competitor achieves a higher profit than the OEM in the competition.

It is commonly accepted without any thorough scientific justification that product cannibalization should be avoided. One of the purposes of this research is to challenge this common belief and to show that product cannibalization, which carries negative allusions, may not be the right terminology to describe the situation where the demand for remanufactured products replaces some demands for new products. We argue that remanufactured products should be considered as one of product categories which makes up the entire product portfolio. From the perspective of revenue management, it is desirable to diversify the product portfolio by offering several different types of products to consumers as this will reach broader market segments, create new demands, and increase profit. However, researchers and practitioners have been focusing on minimizing or avoiding product cannibalization of new products when it comes to 'revenue management' in CLSCs. Our results show that remanufacturing can increase a firm's profit, but product cannibalization of new product by remanufactured product is also likely. We verify this result for several different distributions of consumers' WTP, i.e., different markets, such as normal distributions, two-Gaussian mixtures, and uniform distributions.

We identified two important issues with a solution (i.e., a set of prices) obtained with a uniform assumption for a WTP distribution. First, it may not be possible to implement the solution obtained from a uniform WTP assumption in a real market due to the feasibility issues. Second, companies may suffer from the low profit performance of the approximate solution based on a uniform WTP distribution assumption in a real market. We demonstrated that optimal prices can vary significantly depending on the distribution of consumers' WTP. But a uniform assumption for a real WTP distribution tends to oversimplify a given market characteristics, which gives an infeasible or low-quality solution. Implementing the right prices is important, especially in today's market where consumers are highly price sensitive given the increase in price transparency. Our results show that the knowledge of consumers' WTP distribution is critical for profitable and viable product remanufacturing.

Numerical results are used to demonstrate the application of the new models developed here and to illustrate the kinds of insights that can be shown for specific cases. However, these insights are limited to the particular numerical example studied. For example, we used hypothetical distributions such as normal distributions and gaussian mixtures to model consumers' WTP and WTA. The optimal prices, profit, and the degree of product cannibalization could be different if actual WTP and WTA distributions are used.

CHAPTER V

Contributions and Future Research Directions

5.1 Contributions

This dissertation seeks to advance knowledge and understanding of decision-making in closed-loop supply chains (CLSCs) through new quantitative models.

In Chapter II, we studied joint control of stochastic forward and stochastic reverse material flows. CLSCs involve forward production activities and reverse production activities, but most manufactures separate the operations of the two production systems. With an application to a CLSC where postconsumer products are collected for warranty service purposes, we demonstrated that the benefit of coordinating two production activities could be significant. We developed a model that can be used to obtain an effective inventory control policy for coordinating forward and reverse material flows. Through Monte Carlo simulation and global sensitivity analysis, we identified major influential factors that affect system's warranty cost savings performance. The results indicate that joint control of forward and reverse material flows greatly improves system's robustness to uncertainties as well as warranty cost savings performance.

In Chapter III, we developed a differential game model for characterizing decentralized time-varying competitive decision-making in a CLSC. This is a new approach in the field of CLSC research. Understanding temporal aspects of decentralized decision-making in a CLSC is important for properly adapting the system to today's rapidly changing market environment. Unlike traditional forward supply chains, CLSCs involve many stakeholders who pursue different objectives in forward and reverse production activities. As such, the differential game modeling framework is particularly useful for studying time-varying competitive interactions among decentralized decision makers in CLSCs. We identified optimal prices and production strategies that evolve over time under nonstationary market demand. Also, the model provides quantitative scheme that can be used to obtain an efficient apportionment of product recovery process.

In Chapter IV, we developed models that jointly determine optimal prices in forward and reverse channels considering consumers' WTP for remanufactured products, consumers' WTA for a buyback price, consumers' risk aversion to uncertainty. Whereas price is one of the most effective variables for managing market demand, previous CLSC research has mainly focused on operational problems without paying much attention on the interface between CLSC and market. The imbalance in knowledge has resulted in the fear of product cannibalization. This research attempts to restore a balance in the knowledge of CLSC by addressing important marketing elements in pricing decisions. We obtained new insights on product cannibalization based on quantitative models. The results suggest that the issue of product cannibalization is better addressed by a well-informed model that consolidates operational decisions with information on consumer characteristics. The proposed models reveal detailed interactions among consumers' WTP/WTA, uncertainty in return flows, product cannibalization, and firm's profit.

In summary, this dissertation provides new insights on optimal decision making in CLSCs where forward and reverse production activities are jointly coordinated.

5.2 Future Research Directions

The stochastic models discussed in Chapter II assume stationary demand, whereas the dynamic game model in Chapter III assumes deterministic demand. Although these models provide useful insights, neither case truly represents real situations.

One of ways to improve the model's limitation is to integrate both models within a stochastic differential game. In this way, we can address the issues in Chapter II and Chapter III without needing to assume stationary deterministic demand. This modeling approach will enable knowledge and insights that better explain real situations where various internal and external uncertainties and dynamic interactions among decision-makers exist.

The dynamic game model in Chapter III can be extended in several ways. First, in the discussion we have identified a specific range for the values of ϕ_R and ϕ_M in which the manufacturer is able to lower the optimal wholesale price while engaging in reverse production activities. The reason why we provided only the range on ϕ_R is that a closed form solution for ϕ_R^* is intractable for a general form of $g(\phi_M)$. Nevertheless, we expect that, with some modification to the present approach or to the model, it would be possible and interesting to determine the ϕ_R^* and ϕ_M^* which optimize the performance of the system for a given characteristic curve $g(\phi_M)$ and system parameters. Second, uncertainty in the return flows is of special interest since it significantly complicates the operation of reverse and forward productions in many practical situations. In particular, if the realization of market demands is significantly different from what was expected initially, then the retailer may defect from its initial decision. The manufacturer and retailer should be able to protect themselves from such uncertainties, but the best structure under which to analyze the system behavior is not immediately apparent.

In the duopoly model in Chapter IV, we assumed that the OEM only sells new product but does not collect used products. In practical cases, the OEM may collect its used products in order to prevent other remanufacturing firms from collecting and remanufacturing its used products, which could cannibalize the demand for new product. In this case, the OEM can try to cannibalize the return flows in the reverse channel, while remanufacturing firms try to cannibalize the demand in the forward channel. This problem can be addressed with a slight modification to our current model by implementing a competition mechanism in the reverse channel and assigning one more decision variable, i.e., another buyback price, to the OEM.

Our results indicates that the OEM's profit performance is significantly degraded when it is challenged by competition from a remanufacturing firm. The best case for the OEM happens when it monopolizes the market with its new and remanufactured products. There may exist substantial levels of product cannibalization, yet our results demonstrate that this is much better than competing with a remanufacturing firm. This provides insights to OEM inkjet printer manufacturers. We have not heard of any case where an OEM sells remanufactured inkjet printer cartridges. But there are many inkjet cartridge refillers who compete with OEMs. One way to improve the situation would be to employ a suitable coordination scheme that aligns the objectives of the competing firms. This extension could provide useful insights on setting up the entire system in such a way that all stakeholders including firms and consumers are better off while making the system more sustainable and financially viable.

BIBLIOGRAPHY

Arrow, K., S. Karlin, and H. Scarf (1958), *Studies in the mathematical theory of inventory and production*, chap. Steady State Solutions, Stanford University Press.

Arthur F. Veinott, J. (1965), Optimal policy for a multi-product, dynamic, nonstationary inventory problem, *Management Science*, *12*(3), 206–222.

Atasu, A., M. Sarvary, and L. N. Van Wassenhove (2008), Remanufacturing as a marketing strategy, *Management science*, *54*(10), 1731–1746.

Atasu, A., V. D. R. Guide, Jr., and L. N. Van Wassenhove (2010), So what if remanufacturing cannibalizes my new product sales?, *California Management Review*, *52*(2), 56 – 76.

Athey, S. (2002), Monotone comparative statics under uncertainty, *The Quarterly Journal of Economics*, *117*(1), 187–223.

Bayindir, Z. P., N. Erkip, and R. Gullu (2005), Assessing the benefits of remanufacturing option under one-way substitution, *The Journal of the Operational Research Society*, *56*(3), 286–296.

Bollapragada, S., and T. E. Morton (1999), Myopic heuristics for the random yield problem, *Operations Research*, *47*(5), 713–722.

Breidert, C., M. Hahsler, and T. Reutterer (2006), A review of methods for measuring willingness-to-pay, *Innovative Marketing*, *2*(4), 8–32.

Cachon, G. P. (2003), *Handbooks in operations research and management science: supply chain management*, chap. Supply Chain Coordination with Contracts, pp. 229–340, North-Holland.

Cachon, G. P., and S. Netessine (2004), *Handbook of quantitative supply chain analysis: modeling in the e-business era*, chap. Game theory in supply chain analysis, pp. 13–66, Kluwer Academic Publishers.

Cachon, G. P., T. Randall, and G. M. Schmidt (2007), In search of the bullwhip effect, *Manufacturing & Service Operations Management*, *9*, 457 – 479.

Carpet America Recovery Effort (2008), Carpet america recovery effort 2008 annual report.

Chan, L. M. A., Z. J. M. Shen, D. Simchi-Levi, and J. L. Swann (2004), *International series in operations research and management science*, chap. Coordination of Pricing and Inventory Decisions: A Survey and Classification, pp. 335–392, Kluwer Academic Publishers.

Clark, A. J., and H. Scarf (1960), Optimal policies for a multi-echelon inventory problem, *Management science*, *6*(4), 475–490.

Debo, L. G., L. B. Toktay, and L. N. V. Wassenhove (2005), Market segmentation and product technology selection for remanufacturable products, *Management Science*, *51*(8), 1193–1205.

Debo, L. G., L. B. Toktay, and L. N. Van Wassenhove (2006), Joint life-cycle dynamics of new and remanufactured products, *Production and Operations Management*, *15*(4), 498–513.

DeCroix, G., J.-S. Song, and P. Zipkin (2005), A series system with returns: Stationary analysis, *Operations Research*, *53*(2), 350–362.

DeCroix, G. A. (2006), Optimal policy for a multiechelon inventory system with remanufacturing, *Operations Research*, *54*(3), 532–543.

DeCroix, G. A., and P. H. Zipkin (2005), Inventory management for an assembly system with product or component returns, *Management Science*, *51*(8), 1250–1265.

Duch, R. M. (2010), Risk preferences and vote choice: Experimental insights, university of Oxford.

Electronics TakeBack Coalition (), Facts and figures on ewaste and recycling.

Eliashberg, J., and R. Steinberg (1987), Marketing-production decisions in an industrial channel of distribution, *Management science*, *33*(8), 981–1000.

Environment Policy Committee (2005), Improving recycling markets, *Tech. rep.*, Working Group on Waste Prevention and Recycling, OECD.

Environmental Protection Agency (2010), Statistics on the management of used and end-of-life electronics.

Ferguson, M. E. (2009), Strategic and tactical aspects of closed-loop supply chains, *Foundations and Trends in Technology, Information and Operations Management*, *3*(2), 103–200.

Ferguson, M. E., and L. B. Toktay (2006), The effect of competition on recovery strategies, *Production and Operations Management*, *15*(3), 351–368.

Fleischmann, M., R. Kuik, and R. Dekker (2002), Controlling inventories with stochastic item returns: A basic model, *European Journal of Operational Research*, *138*(1), 63 – 75.

Gerchak, Y., and M. Henig (1989), Component commonality in assemble-to-order systems: Models and properties, *Naval Research Logistics*, *36*(1), 61–68.

Guide, V. D. R., Jr. (2000), Production planning and control for remanufacturing: industry practice and research needs, *Journal of Operations Management*, *18*(4), 467–483.

Guide, V. D. R., Jr., and L. N. Van Wassenhove (2009), The evolution of closed-loop supply chain research, *Operations Research*, *57*(1), 10–18.

Guide, V. D. R., Jr., R. H. Teunter, and L. N. Van Wassenhove (2003), Matching demand and supply to maximize profits from remanufacturing, *Manufacturing & Service Operations Management*, *5*(4), 303–316.

Gungor, A., and S. M. Gupta (1999), Issues in environmentally conscious manufacturing and product recovery: a survey, *Computers & Industrial Engineering*, *36*(4), 811–853.

Hartl, R. F. (1995), Production smoothing under environmental constraints, *Production and Operations Management*, *4*(1), 46–56.

He, X., A. Prasad, S. P. Sethi, and G. J. Gutierrez (2007), A survey of differential game models in supply and marketing channels, *Journal of Systems Science and Systems Engineering*, *16*(4), 385–413.

Hewlett-Packard Annual Report (2010), 2010 hp annual report.

Huang, J.-J., G.-H. Tzeng, and C.-S. Ong (2007), Marketing segmentation using support vector clustering, *Expert Systems with Applications*, *32*(2), 313 – 317.

Huang, W., V. Kulkarni, and J. M. Swaminathan (2008), Managing the inventory of an item with a replacement warranty, *Management Science*, *54*(8), 1441–1452.

Huh, W. T., and M. Nagarajan (2010), Technical note–linear inflation rules for the random yield problem: Analysis and computations, *Operations Research*, *58*(1), 244–251.

Ilgin, M. A., and S. M. Gupta (2010), Environmentally conscious manufacturing and product recovery (ecmpro): A review of the state of the art, *Journal of Environmental Management*, *91*(3), 563 – 591.

Inderfurth, K. (1997), Simple optimal replenishment and disposal policies for a product recovery system with leadtimes, *OR Spectrum*, *19*(2), 111–122.

Inderfurth, K., and S. Transchel (2007), Technical note – note on "myopic heuristics for the random yield problem", *Operations Research*, *55*(6), 1183–1186.

Johnson, G. D., and H. E. Thompson (1975), Optimality of myopic inventory policies for certain dependent demand processes, *Management Science*, *21*(11), 1303 – 1307.

Khawam, J., W. H. Hausman, and D. W. Cheng (2007), Warranty inventory optimization for hitachi global storage technologies, inc., *Interfaces*, *37*(5), 455–471.

Khawam, J. H. (2009), Warranty inventory management and supplier decision models, Ph.D. thesis, Stanford University.

Kiesmüller, G., R. Kleber, and S. Minner (2004), *Reverse Logistics: Quantitative Models for Closed-loop Supply Chains*, chap. Managing dynamic product recovery: an optimal control perspective, Springer.

Kiesmüller, G. P., and S. Minner (2003), Simple expressions for finding recovery system inventory control parameter values, *Journal of the Operational Research Society*, *54*, 83–88.

Klausner, M., and C. T. Hendrickson (2000), Reverse-logistics strategy for product take-back, *Interfaces*, *30*(3), 156–165.

Knowles, G. (1981), *An Introduction to Applied Optimal Control*, Academic Press.

Kodak (2007), 2007 global sustainability annual report.

Kumazawa, T., and H. Kobayashi (2006), A simulation system to support the establishment of circulated business, *Advanced Engineering Informatics*, *20*(2), 127–136.

Lee, C., M. Realff, and J. Ammons (2011), Integration of channel decisions in a decentralized reverse production system with retailer collection under deterministic non-stationary demands, *Advanced Engineering Informatics*, *25*(1), 88–102.

Lee, E., and R. Staelin (1997), Vertical strategic interaction: Implications for channel pricing strategy, *Marketing Science*, *16*(3), 185–207.

Li, Y., J. B. Richardson, R. M. Bricka, X. Niu, H. Yang, L. Li, and A. Jimenez (2009), Leaching of heavy metals from e-waste in simulated landfill columns, *Waste Management*, *29*(7), 2147 – 2150.

Majumder, P., and H. Groenevelt (2001), Competition in remanufacturing, *Production and Operations Management*, *10*(2), 125–141.

Maslennikova, I., and D. Foley (2000), Xerox's approach to sustainability, *Interfaces*, *30*(3), 226–233.

Milgrom, P. (1994), Comparing optima: Do simplifying assumptions affect conclusions?, *The Journal of Political Economy*, *102*(3), 607–615.

Minner, S., E. A. Silver, and D. J. Robb (2003), An improved heuristic for deciding on emergency transshipments, *European Journal of Operational Research*, *148*(2), 384 – 400.

Mitra, S. (2007), Revenue management for remanufactured products, *Omega*, *35*(5), 553–562.

Mladenovic, N., and P. Hansen (1997), Variable neighborhood search, *Computers & Operations Research*, *24*(11), 1097 – 1100.

Moorthy, K. S., and I. P. L. Png (1992), Market segmentation, cannibalization, and the timing of product introductions, *Management Science*, *38*(3), 345–359.

Murthy, D. N. P., and W. R. Blischke (1992), Product warranty management – iii: A review of mathematical models, *European Journal of Operational Research*, *63*(1), 1–34.

Murthy, D. N. P., and I. Djamaludin (2002), New product warranty: A literature review, *International Journal of Production Economics*, *79*(3), 231 – 260.

Oakdene Hollins (2007), Project update 2007/08.

Okada, E. M. (2010), Uncertainty, risk aversion, and wta vs. wtp, *Marketing Science*, *29*(1), 75–84.

Ovchinnikov, A. (2011), Revenue and cost management for remanufactured products, *Production and Operations Management*.

Pearce, J. A. (2009), The profit-making allure of product reconstruction, *Tech. rep.*, MIT Sloan Management Review.

Pedroso, J. P. (1996), Numerical solution of nash and stackelberg equilibria: an evolutionary approach, in *Proceedings of the First Asia Conference on Simulated Evolution and Learning, Korea*.

Pekelman, D. (1974), Simultaneous price-production decisions, *Operations Research*, *22*(4), 788–794.

Pontryagin, L., V. Boltyanskii, R. Gamkrelidze, and E. Mischenko (1962), *The mathematical theory of optimal processes*, Interscience.

Robinson, L. W. (1990), Optimal and approximate policies in multiperiod, multilocation inventory models with transshipments, *Operations Research*, *38*(2), 278–295.

Rudi, N., S. Kapur, and D. F. Pyke (2001), A two-location inventory model with transshipment and local decision making, *Management Science*, *47*(12), 1668–1680.

Saltelli, A. (2002), Making best use of model evaluations to compute sensitivity indices, *Computer Physics Communications*, *145*(2), 280 – 297.

Saltelli, A., M. Ratto, T. Andres, F. Campolongo, J. Cariboni, D. Gatelli, M. Saisana, and S. Tarantola (2008), *Global Sensitivity Analysis: The Primer*, John Wiley & Sons Inc.

Savaskan, R. C., S. Bhattacharya, and L. N. Van Wassenhove (2004), Closed-loop supply chain models with product remanufacturing, *Management science*, *50*(2), 239–252.

Shioda, R., L. Tunçel, and T. Myklebust (2011), Maximum utility product pricing models and algorithms based on reservation price, *Computational Optimization and Applications*, *48*, 157–198.

Simpson, V. P. (1978), Optimum solution structure for a repairable inventory problem, *Operations Research*, *26*(2), 270–281.

Sobol, I. M. (2001), Global sensitivity indices for nonlinear mathematical models and their monte carlo estimates, *Mathematics and Computers in Simulation*, *55*(1-3), 271–280.

Spence, M. (1977), Consumer misperceptions, product failure and producer liability, *The Review of Economic Studies*, *44*(3), 561–572.

The University of Illinois at Urbana-Champaign Sustainable Technology Center (2009), Strategies for improving the sustainability of e-waste management systems, *Tech. rep.*

U.S. Geological Survey (2001), U.s. geological survey fact sheet 060-01.

van der Laan, E., M. Salomon, R. Dekker, and L. Van Wassenhove (1999), Inventory control in hybrid systems with remanufacturing, *Management science*, *45*(5), 733–747.

Voelckner, F. (2006), An empirical comparison of methods for measuring consumers' willingness to pay, *Marketing Letters*, *17*(2), 137–149.

Yano, C. A., and H. L. Lee (1995), Lot sizing with random yields: A review, *Operations Research*, *43*(2), 311–334.